

**irf create** — Obtain IRFs, dynamic-multiplier functions, and FEVDs

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## Description

`irf create` estimates multiple sets of impulse–response functions (IRFs), dynamic-multiplier functions, and forecast-error variance decompositions (FEVDs). All of these estimates and their standard errors are known collectively as IRF results and are saved in an IRF file under a specified filename. Once you have created a set of IRF results, you can use the other [irf](#) commands to analyze them.

## Quick start

Create impulse–response function `myirf` with 8 forecast periods in the active IRF file

```
irf create myirf
```

Same as above, and use IRF file `myirfs.irf`

```
irf create myirf, set(myirfs)
```

Same as above, but compute the IRF for 12 periods

```
irf create myirf, set(myirfs) step(12)
```

Note: `irf` commands can be used after `var`, `svar`, `ivsvar`, `vec`, `arima`, `arfima`, `dsge`, or `dsgenl`; see [\[TS\] var](#), [\[TS\] var svar](#), [\[TS\] var ivsvar](#), [\[TS\] vec](#), [\[TS\] arima](#), [\[TS\] arfima](#), [\[DSGE\] dsge](#), or [\[DSGE\] dsgenl](#).

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## Syntax

After `var`

```
irf create irfname [ , var_options ]
```

After `svar`

```
irf create irfname [ , svar_options ]
```

After `ivsvar` (*StataNow*)

```
irf create irfname [ , ivsvvar_options ]
```

After `vec`, `dsge`, or `dsge1`

```
irf create irfname [ , vec_options ]
```

After `arima`

```
irf create irfname [ , arima_options ]
```

After `arfima`

```
irf create irfname [ , arfima_options ]
```

*irfname* is any valid name that does not exceed 15 characters.

<i>var_options</i>	Description
Main	
<code>set(<i>filename</i> [ , <i>replace</i> ])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>order(<i>varlist</i>)</code>	specify Cholesky ordering of endogenous variables
<code>estimates(<i>estname</i>)</code>	use previously stored results <i>estname</i> ; default is to use active results
Std. errors	
<code>nose</code>	do not calculate standard errors
<code>bs</code>	obtain standard errors from bootstrapped residuals
<code>bsp</code>	obtain standard errors from parametric bootstrap
<code>nodots</code>	do not display “.” for each bootstrap replication
<code>reps(#)</code>	use # bootstrap replications; default is <code>reps(200)</code>
<code>bsaving(<i>filename</i> [ , <i>replace</i> ])</code>	save bootstrap results in <i>filename</i>

<i>svar_options</i>	Description
Main	
<code>set(filename[, replace])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>estimates(estname)</code>	use previously stored results <i>estname</i> ; default is to use active results
Std. errors	
<code>nose</code>	do not calculate standard errors
<code>bs</code>	obtain standard errors from bootstrapped residual
<code>bsp</code>	obtain standard errors from parametric bootstrap
<code>nodots</code>	do not display “.” for each bootstrap replication
<code>reps(#)</code>	use # bootstrap replications; default is <code>reps(200)</code>
<code>bsaving(filename[, replace])</code>	save bootstrap results in <i>filename</i>

<i>ivsvar_options</i>	Description
Main	
<code>set(filename[, replace])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>estimates(estname)</code>	use previously stored results <i>estname</i> ; default is to use active results

<i>vec_options</i>	Description
Main	
<code>set(filename[, replace])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>estimates(estname)</code>	use previously stored results <i>estname</i> ; default is to use active results

<i>arima_options</i>	Description
Main	
<code>set(filename[, replace])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>estimates(estname)</code>	use previously stored results <i>estname</i> ; default is to use active results
Std. errors	
<code>nose</code>	do not calculate standard errors

---

<i>arfima_options</i>	Description
Main	
<code>set(filename[, replace])</code>	make <i>filename</i> active
<code>replace</code>	replace <i>irfname</i> if it already exists
<code>step(#)</code>	set forecast horizon to #; default is <code>step(8)</code>
<code>smemory</code>	calculate short-memory IRFs
<code>estimates(estname)</code>	use previously stored results <i>estname</i> ; default is to use active results
Std. errors	
<code>nose</code>	do not calculate standard errors

The default is to use asymptotic standard errors if no options are specified.

`irf create` is for use after fitting a model with the `var`, `svar`, `ivsvar`, `vec`, `arma`, `arfima`, `lpirf`, `dsge`, or `dsgenl` command; see [TS] `var`, [TS] `var svar`, [TS] `var ivsvar`, [TS] `vec`, [TS] `arma`, [TS] `arfima`, [TS] `lpirf`, [DSGE] `dsge`, or [DSGE] `dsgenl`.

You must `tsset` your data before using `var`, `svar`, `ivsvar`, `vec`, `arma`, `arfima`, `lpirf`, `dsge`, or `dsgenl` and, hence, before using `irf create`; see [TS] `tsset`.

## Options

### Main

`set(filename[, replace])` specifies the IRF file to be used. If `set()` is not specified, the active IRF file is used; see [TS] `irf set`.

If `set()` is specified, the specified file becomes the active file, just as if you had issued an `irf set` command.

`replace` specifies that the results saved under *irfname* may be replaced, if they already exist. IRF results are saved in files, and one file may contain multiple IRF results.

`step(#)` specifies the step (forecast) horizon; the default is eight periods.

`order(varlist)` is allowed only after estimation by `var`; it specifies the Cholesky ordering of the endogenous variables to be used when estimating the orthogonalized IRFs. By default, the order in which the variables were originally specified on the `var` command is used.

`smemory` is allowed only after estimation by `arfima`; it specifies that the IRFs are calculated based on a short-memory model with the fractional difference parameter  $d$  set to zero.

`estimates(estname)` specifies that estimation results previously estimated by `var`, `svar`, `ivsvar`, or `vec`, and stored by `estimates`, be used. This option is rarely specified; see [R] `estimates`.

### Std. errors

`nose`, `bs`, and `bsp` are alternatives that specify how (whether) standard errors are to be calculated. If none of these options is specified, asymptotic standard errors are calculated, except in two cases: after estimation by `vec` and after estimation by `svar` in which long-run constraints were applied. In those two cases, the default is as if `nose` were specified, although in the second case, you could specify `bs` or `bsp`. After estimation by `vec`, standard errors are simply not available.

`nose` specifies that no standard errors be calculated.

`bs` specifies that standard errors be calculated by bootstrapping the residuals. `bs` may not be specified if there are gaps in the data.

bsp specifies that standard errors be calculated via a multivariate-normal parametric bootstrap. bsp may not be specified if there are gaps in the data.

nodots, reps(#), and bsave(filename[, replace]) are relevant only if bs or bsp is specified.

nodots specifies that dots not be displayed each time irf create performs a bootstrap replication. reps(#), # > 50, specifies the number of bootstrap replications to be performed. reps(200) is the default.

bsave(filename[, replace]) specifies that file filename be created and that the bootstrap replications be saved in it. New file filename is just a .dta dataset that can be loaded later using use; see [D] use. If filename is specified without an extension, .dta is assumed.

## Remarks and examples

[stata.com](http://www.stata.com)

If you have not read [TS] irf, please do so. An introductory example using IRFs is presented there.

irf create estimates several types of IRFs, dynamic-multiplier functions, and FEVDs. Which estimates are saved depends on the estimation method previously used to fit the model, as summarized in the table below:

Saves	Estimation command						
	arima/ arfima	var	svar	ivsvar <sup>†</sup>	vec	dsge/ dsgenl	lpirf
simple IRFs	x	x	x	x	x	x	x
orthogonalized IRFs	x	x	x	x	x		x
dynamic multipliers	x	x					
cumulative IRFs	x	x	x	x	x		
cumulative orthogonalized IRFs	x	x	x	x	x		
cumulative dynamic multipliers		x					
structural IRFs	x		x	x			
Cholesky FEVDs		x	x		x		
structural FEVDs			x				

<sup>†</sup> ivsvar is part of [StataNow](http://www.stata.com).

Remarks are presented under the following headings:

*Introductory examples*

*Technical aspects of IRF files*

*IRFs and FEVDs*

*IRF results for VAR models*

*An introduction to impulse–response functions for VAR models*

*An introduction to dynamic-multiplier functions for VAR models*

*An introduction to forecast-error variance decompositions for VAR models*

*IRF results for VEC models*

*An introduction to impulse–response functions for VEC models*

*An introduction to forecast-error variance decompositions for VEC models*

*IRF results for ARIMA and ARFIMA*

## Introductory examples

## ▷ Example 1: After var

Below we compare bootstrap and asymptotic standard errors for a specific FEVD. We begin by fitting a VAR(2) model to the Lütkepohl data (we use the `var` command). We next use the `irf create` command twice, first to create results with asymptotic standard errors (saved under the name `asympt`) and then to re-create the same results, this time with bootstrap standard errors (saved under the name `bs`). Because bootstrapping is a random process, we set the random-number seed (`set seed 123456`) before using `irf create` the second time; this makes our results reproducible. Finally, we compare results by using the IRF analysis command `irf ctable`.

```
. use https://www.stata-press.com/data/r18/lutkepohl2
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)
. var dln_inv dln_inc dln_consump if qtr>=tq(1961q2) & qtr<=tq(1978q4), lags(1/2)
(output omitted)
. irf create asympt, step(8) set(results1)
(file results1.irf created)
(file results1.irf now active)
(file results1.irf updated)
. set seed 123456
. irf create bs, step(8) bs reps(250) nodots
(file results1.irf updated)
. irf ctable (asympt dln_inc dln_consump fevd) (bs dln_inc dln_consump fevd),
> noci stderr
```

Step	(1) fevd	(1) S.E.	(2) fevd	(2) S.E.
0	0	0	0	0
1	.282135	.087373	.282135	.102756
2	.278777	.083782	.278777	.098161
3	.33855	.090006	.33855	.10586
4	.339942	.089207	.339942	.104191
5	.342813	.090494	.342813	.105351
6	.343119	.090517	.343119	.105258
7	.343079	.090499	.343079	.105266
8	.34315	.090569	.34315	.105303

(1) `irfname = asympt`, `impulse = dln_inc`, and `response = dln_consump`.  
(2) `irfname = bs`, `impulse = dln_inc`, and `response = dln_consump`.

Point estimates are, of course, the same. The bootstrap estimates of the standard errors, however, are larger than the asymptotic estimates, which suggests that the sample size of 71 is not large enough for the distribution of the estimator of the FEVD to be well approximated by the asymptotic distribution. Here we would expect the bootstrap confidence interval to be more reliable than the confidence interval that is based on the asymptotic standard error.

## □ Technical note

The details of the bootstrap algorithms are given in *Methods and formulas*. These algorithms are conditional on the first  $p$  observations, where  $p$  is the order of the fitted vector autoregressive (VAR) model. (In a structural VAR (SVAR) model,  $p$  is the order of the VAR model that underlies the SVAR model.) The bootstrapped estimates are conditional on the first  $p$  observations, just as the estimators of the coefficients in VAR models are conditional on the first  $p$  observations. With bootstrap standard errors (option `bs`), the  $p$  initial observations are used with resampling the residuals to produce the bootstrap samples used for estimation. With the more parametric bootstrap (option `bsp`), the  $p$  initial observations are used with draws from a multivariate normal distribution with variance–covariance matrix  $\widehat{\Sigma}$  to generate the bootstrap samples. □

## □ Technical note

For `var` and `svar` `e()` results, `irf` uses  $\widehat{\Sigma}$ , the estimated variance matrix of the disturbances, in computing the asymptotic standard errors of all the functions. The point estimates of the orthogonalized impulse–response functions, the structural impulse–response functions, and all the variance decompositions also depend on  $\widehat{\Sigma}$ . As discussed in [TS] `var`, `var` and `svar` use the ML estimator of this matrix by default, but they have option `dfk`, which will instead use an estimator that includes a small-sample correction. Specifying `dfk` when the model is fit—when the `var` or `svar` command is given—changes the estimate of  $\widehat{\Sigma}$  and will change the IRF results that depend on it. □

## ▷ Example 2: After var with exogenous variables

After fitting a VAR model, `irf create` computes estimates of the dynamic multipliers, which describe the impact of a unit change in an exogenous variable on each endogenous variable. For instance, below we estimate and report the cumulative dynamic multipliers from a model in which changes in investment are exogenous. The results indicate that both of the cumulative dynamic multipliers are significant.

```
. var dln_inc dln_consump if qtr>=tq(1961q2) & qtr<=tq(1978q4), lags(1/2)
> exog(L(0/2).dln_inv)
(output omitted)
. irf create dm, step(8)
(file results1.irf updated)
. irf table cdm, impulse(dln_inv) irf(dm)
Results from dm
```

Step	(1) cdm	(1) Lower	(1) Upper
0	.032164	-.027215	.091544
1	.096568	.003479	.189656
2	.140107	.022897	.257317
3	.150527	.032116	.268938
4	.148979	.031939	.26602
5	.151247	.033011	.269482
6	.150267	.033202	.267331
7	.150336	.032858	.267813
8	.150525	.033103	.267948

Step	(2) cdm	(2) Lower	(2) Upper
0	.058681	.012529	.104832
1	.062723	-.005058	.130504
2	.126167	.032497	.219837
3	.136583	.038691	.234476
4	.146482	.04442	.248543
5	.146075	.045201	.24695
6	.145542	.044988	.246096
7	.146309	.045315	.247304
8	.145786	.045206	.246365

95% lower and upper bounds reported.

(1) irfname = dm, impulse = dln\_inv, and response = dln\_inc.

(2) irfname = dm, impulse = dln\_inv, and response = dln\_consump.

### ► Example 3: After vec

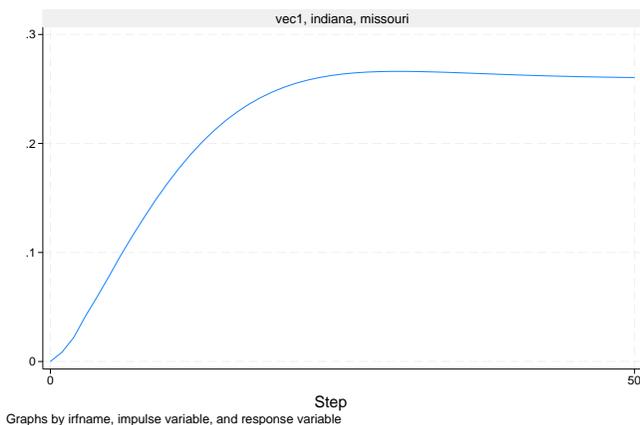
Although all IRFs and orthogonalized IRFs (OIRFs) from models with stationary variables will taper off to zero, some of the IRFs and OIRFs from models with first-difference stationary variables will not. This is the key difference between IRFs and OIRFs from systems of stationary variables fit by `var` or `svar` and those obtained from systems of first-difference stationary variables fit by `vec`. When the effect of the innovations dies out over time, the shocks are said to be transitory. In contrast, when the effect does not taper off, shocks are said to be permanent.

In this example, we look at the OIRF from one of the vector error-correction (VEC) models fit to the unemployment-rate data analyzed in [example 2](#) of [\[TS\] vec](#). We see that an orthogonalized shock to Indiana has a permanent effect on the unemployment rate in Missouri:

```
. use https://www.stata-press.com/data/r18/urates
. vec missouri indiana kentucky illinois, trend(rconstant) rank(2) lags(4)
  (output omitted)
. irf create vec1, set(vecirfs) step(50)
(file vecirfs.irf created)
(file vecirfs.irf now active)
(file vecirfs.irf updated)
```

Now we can use `irf graph` to graph the OIRF of interest:

```
. irf graph oirf, impulse(indiana) response(missouri)
```



The graph shows that the estimated OIRF converges to a positive asymptote, which indicates that an orthogonalized innovation to the unemployment rate in Indiana has a permanent effect on the unemployment rate in Missouri.

## Technical aspects of IRF files

This section is included for programmers wishing to extend the `irf` system.

`irf create` estimates a series of impulse–response functions and their standard errors. Although these estimates are saved in an IRF file, most users will never need to look at the contents of this file. The IRF commands `fill in`, `analyze`, `present`, and `manage` IRF results.

IRF files are just Stata datasets that have names ending in `.irf` instead of `.dta`. The dataset in the file has a nested panel structure.

Variable `irfname` contains the *irfname* specified by the user. Variable `impulse` records the name of the endogenous variable whose innovations are the impulse. Variable `response` records the name of the endogenous variable that is responding to the innovations. In a model with  $K$  endogenous variables, there are  $K^2$  combinations of `impulse` and `response`. Variable `step` records the periods for which these estimates were computed.

Below is a catalog of the statistics that `irf create` estimates and the variable names under which they are saved in the IRF file.

Statistic	Name
impulse–response functions	<code>irf</code>
orthogonalized impulse–response functions	<code>oirf</code>
dynamic-multiplier functions	<code>dm</code>
cumulative impulse–response functions	<code>cirf</code>
cumulative orthogonalized impulse–response functions	<code>coirf</code>
cumulative dynamic-multiplier functions	<code>cdm</code>
Cholesky forecast-error decomposition	<code>fevd</code>
structural impulse–response functions	<code>sirf</code>
structural forecast-error decomposition	<code>sfevd</code>
standard error of the impulse–response functions	<code>stdirf</code>
standard error of the orthogonalized impulse–response functions	<code>stdoirf</code>
standard error of the cumulative impulse–response functions	<code>stdcirf</code>
standard error of the cumulative orthogonalized impulse–response functions	<code>stdcoirf</code>
standard error of the Cholesky forecast-error decomposition	<code>stdfevd</code>
standard error of the structural impulse–response functions	<code>stdsirf</code>
standard error of the structural forecast-error decomposition	<code>stdsfevd</code>

In addition to the variables, information is stored in `_dta` characteristics. Much of the following information is also available in `r()` after `irf describe`, where it is often more convenient to obtain the information. Characteristic `_dta[version]` contains the version number of the IRF file, which is currently 1.1. Characteristic `_dta[irfnames]` contains a list of all the *irfnames* in the IRF file. For each *irfname*, there are a series of additional characteristics:

Name	Contents
<code>_dta[irfname_model]</code>	<code>var</code> , <code>sr var</code> , <code>lr var</code> , <code>ivsvar</code> , <code>vec</code> , <code>lpirf</code> , <code>arma</code> , <code>arfima</code> , <code>dsge</code> , or <code>dsgenl</code>
<code>_dta[irfname_order]</code>	Cholesky order used in IRF estimates
<code>_dta[irfname_exog]</code>	exogenous variables, and their lags, in VAR models
<code>_dta[irfname_exogvars]</code>	exogenous variables in VAR models
<code>_dta[irfname_constant]</code>	<code>constant</code> or <code>noconstant</code> , depending on whether <code>noconstant</code> was specified in <code>var</code> or <code>svar</code>
<code>_dta[irfname_lags]</code>	lags in model
<code>_dta[irfname_exlags]</code>	lags of exogenous variables in model
<code>_dta[irfname_tmin]</code>	minimum value of <code>timevar</code> in the estimation sample
<code>_dta[irfname_tmax]</code>	maximum value of <code>timevar</code> in the estimation sample
<code>_dta[irfname_timevar]</code>	name of <code>tsset</code> <code>timevar</code>
<code>_dta[irfname_tsfmt]</code>	format of <code>timevar</code>
<code>_dta[irfname_varcns]</code>	<code>constrained</code> or colon-separated list of constraints placed on VAR coefficients
<code>_dta[irfname_svarcns]</code>	<code>constrained</code> or colon-separated list of constraints placed on VAR coefficients
<code>_dta[irfname_step]</code>	maximum step in IRF estimates
<code>_dta[irfname_stderror]</code>	<code>asymptotic</code> , <code>bs</code> , <code>bsp</code> , or <code>none</code> , depending on the type of standard errors requested
<code>_dta[irfname_reps]</code>	number of bootstrap replications performed
<code>_dta[irfname_version]</code>	version of the IRF file that originally held <i>irfname</i> IRF results
<code>_dta[irfname_rank]</code>	number of cointegrating equations
<code>_dta[irfname_trend]</code>	<code>trend()</code> specified in <code>vec</code>
<code>_dta[irfname_veccns]</code>	constraints placed on VEC model parameters
<code>_dta[irfname_sind]</code>	normalized seasonal indicators included in <code>vec</code>
<code>_dta[irfname_d]</code>	fractional difference parameter <i>d</i> in <code>arfima</code>

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## IRFs and FEVDs

`irf create` can estimate several types of IRFs and FEVDs for VAR models and VEC models. `irf create` can also estimate IRFs and cumulative IRFs for ARIMA and ARFIMA models. We first discuss IRF results for VAR models and SVAR models, and then we discuss them in the context of VEC models. Because the cointegrating VEC model is an extension of the stationary VAR framework, the section that discusses the IRF results for VEC models draws on the earlier VAR material. We conclude our discussion with IRF results for ARIMA and ARFIMA models.

## IRF results for VAR models

### An introduction to impulse–response functions for VAR models

A  $p$ th-order VAR model with exogenous variables is given by

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B} \mathbf{x}_t + \mathbf{u}_t$$

where

$\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$  is a  $K \times 1$  random vector,  
the  $\mathbf{A}_i$  are fixed  $K \times K$  matrices of parameters,  
 $\mathbf{x}_t$  is an  $R_0 \times 1$  vector of exogenous variables,  
 $\mathbf{B}$  is a  $K \times R_0$  matrix of coefficients,  
 $\mathbf{v}$  is a  $K \times 1$  vector of fixed parameters, and  
 $\mathbf{u}_t$  is assumed to be white noise; that is,

$$\begin{aligned} E(\mathbf{u}_t) &= \mathbf{0} \\ E(\mathbf{u}_t \mathbf{u}_t') &= \Sigma \\ E(\mathbf{u}_t \mathbf{u}_s') &= \mathbf{0} \text{ for } t \neq s \end{aligned}$$

As discussed in [TS] [varstable](#), a VAR model can be rewritten in moving-average form only if it is stable. Any exogenous variables are assumed to be covariance stationary. Because the functions of interest in this section depend only on the exogenous variables through their effect on the estimated  $\mathbf{A}_i$ , we can simplify the notation by dropping them from the analysis. All the formulas given below still apply, although the  $\mathbf{A}_i$  are estimated jointly with  $\mathbf{B}$  on the exogenous variables.

Below we discuss conditions under which the IRFs and forecast-error variance decompositions have a causal interpretation. Although estimation requires only that the exogenous variables be predetermined, that is, that  $E(\mathbf{x}_{jt} u_{it}) = 0$  for all  $i, j$ , and  $t$ , assigning a causal interpretation to IRFs and FEVDs requires that the exogenous variables be strictly exogenous, that is, that  $E(\mathbf{x}_{js} u_{it}) = 0$  for all  $i, j, s$ , and  $t$ .

IRFs describe how the innovations to one variable affect another variable after a given number of periods. For an example of how IRFs are interpreted, see [Stock and Watson \(2001\)](#). They use IRFs to investigate the effect of surprise shocks to the Federal Funds rate on inflation and unemployment. In another example, [Christiano, Eichenbaum, and Evans \(1999\)](#) use IRFs to investigate how shocks to monetary policy affect other macroeconomic variables.

Consider a VAR model without exogenous variables:

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{u}_t \tag{1}$$

The VAR model represents the variables in  $\mathbf{y}_t$  as functions of its own lags and serially uncorrelated innovations  $\mathbf{u}_t$ . All the information about contemporaneous correlations among the  $K$  variables in  $\mathbf{y}_t$  is contained in  $\Sigma$ . In fact, as discussed in [TS] [var svar](#), a VAR model can be viewed as the reduced form of a dynamic simultaneous-equation model.

To see how the innovations affect the variables in  $\mathbf{y}_t$  after, say,  $i$  periods, rewrite the model in its moving-average form

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{i=0}^{\infty} \boldsymbol{\Phi}_i \mathbf{u}_{t-i} \tag{2}$$

where  $\boldsymbol{\mu}$  is the  $K \times 1$  time-invariant mean of  $\mathbf{y}_t$ , and

$$\boldsymbol{\Phi}_i = \begin{cases} \mathbf{I}_K & \text{if } i = 0 \\ \sum_{j=1}^i \boldsymbol{\Phi}_{i-j} \mathbf{A}_j & \text{if } i = 1, 2, \dots \end{cases}$$

We can rewrite a VAR model in the moving-average form only if it is stable. Essentially, a VAR model is stable if the variables are covariance stationary and none of the autocorrelations are too high (the issue of stability is discussed in greater detail in [TS] [varstable](#)).

The  $\Phi_i$  are the simple IRFs. The  $j, k$  element of  $\Phi_i$  gives the effect of a 1-time unit increase in the  $k$ th element of  $\mathbf{u}_t$  on the  $j$ th element of  $\mathbf{y}_t$  after  $i$  periods, holding everything else constant. Unfortunately, these effects have no causal interpretation, which would require us to be able to answer the question, “How does an innovation to variable  $k$ , holding everything else constant, affect variable  $j$  after  $i$  periods?” Because the  $\mathbf{u}_t$  are contemporaneously correlated, we cannot assume that everything else is held constant. Contemporaneous correlation among the  $\mathbf{u}_t$  implies that a shock to one variable is likely to be accompanied by shocks to some of the other variables, so it does not make sense to shock one variable and hold everything else constant. For this reason, (2) cannot provide a causal interpretation.

This shortcoming may be overcome by rewriting (2) in terms of mutually uncorrelated innovations. Suppose that we had a matrix  $\mathbf{P}$ , such that  $\Sigma = \mathbf{P}\mathbf{P}'$ . If we had such a  $\mathbf{P}$ , then  $\mathbf{P}^{-1}\Sigma\mathbf{P}'^{-1} = \mathbf{I}_K$ , and

$$E\{\mathbf{P}^{-1}\mathbf{u}_t(\mathbf{P}^{-1}\mathbf{u}_t)'\} = \mathbf{P}^{-1}E\{(\mathbf{u}_t\mathbf{u}_t')\mathbf{P}'^{-1}\} = \mathbf{P}^{-1}\Sigma\mathbf{P}'^{-1} = \mathbf{I}_K$$

We can thus use  $\mathbf{P}^{-1}$  to orthogonalize the  $\mathbf{u}_t$  and rewrite (2) as

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\mu} + \sum_{i=0}^{\infty} \Phi_i \mathbf{P}\mathbf{P}^{-1}\mathbf{u}_{t-i} \\ &= \boldsymbol{\mu} + \sum_{i=0}^{\infty} \Theta_i \mathbf{P}^{-1}\mathbf{u}_{t-i} \\ &= \boldsymbol{\mu} + \sum_{i=0}^{\infty} \Theta_i \mathbf{w}_{t-i} \end{aligned}$$

where  $\Theta_i = \Phi_i\mathbf{P}$  and  $\mathbf{w}_t = \mathbf{P}^{-1}\mathbf{u}_t$ . If we had such a  $\mathbf{P}$ , the  $\mathbf{w}_k$  would be mutually orthogonal, and no information would be lost in the holding-everything-else-constant assumption, implying that the  $\Theta_i$  would have the causal interpretation that we seek.

Choosing a  $\mathbf{P}$  is similar to placing identification restrictions on a system of dynamic simultaneous equations. The simple IRFs do not identify the causal relationships that we wish to analyze. Thus we seek at least as many identification restrictions as necessary to identify the causal IRFs.

So, where do we get such a  $\mathbf{P}$ ? Sims (1980) popularized the method of choosing  $\mathbf{P}$  to be the Cholesky decomposition of  $\widehat{\Sigma}$ . The IRFs based on this choice of  $\mathbf{P}$  are known as the *orthogonalized IRFs*. Choosing  $\mathbf{P}$  to be the Cholesky decomposition of  $\widehat{\Sigma}$  is equivalent to imposing a recursive structure for the corresponding dynamic structural equation model. The ordering of the recursive structure is the same as the ordering imposed in the Cholesky decomposition. Because this choice is arbitrary, some researchers will look at the OIRFs with different orderings assumed in the Cholesky decomposition. The `order()` option available with `irf create` facilitates this type of analysis.

The SVAR model approach integrates the need to identify the causal IRFs into the model specification and estimation process. Sufficient identification restrictions can be obtained by placing either short-run or long-run restrictions on the model. The VAR model in (1) can be rewritten as

$$\mathbf{y}_t - \mathbf{v} - \mathbf{A}_1\mathbf{y}_{t-1} - \cdots - \mathbf{A}_p\mathbf{y}_{t-p} = \mathbf{u}_t$$

Similarly, a short-run SVAR model can be written as

$$\mathbf{A}(\mathbf{y}_t - \mathbf{v} - \mathbf{A}_1\mathbf{y}_{t-1} - \cdots - \mathbf{A}_p\mathbf{y}_{t-p}) = \mathbf{A}\mathbf{u}_t = \mathbf{B}\mathbf{e}_t \quad (3)$$

where  $\mathbf{A}$  and  $\mathbf{B}$  are  $K \times K$  nonsingular matrices of parameters to be estimated,  $\mathbf{e}_t$  is a  $K \times 1$  vector of disturbances with  $\mathbf{e}_t \sim N(\mathbf{0}, \mathbf{I}_K)$ , and  $E(\mathbf{e}_t \mathbf{e}_s') = \mathbf{0}_K$  for all  $s \neq t$ . Sufficient constraints must be placed on  $\mathbf{A}$  and  $\mathbf{B}$  so that  $\mathbf{P}$  is identified. One way to see the connection is to draw out the implications of the latter equality in (3). From (3), it can be shown that

$$\boldsymbol{\Sigma} = \mathbf{A}^{-1} \mathbf{B} (\mathbf{A}^{-1} \mathbf{B})'$$

As discussed in [TS] `var svar`, the estimates  $\widehat{\mathbf{A}}$  and  $\widehat{\mathbf{B}}$  are obtained by maximizing the concentrated log-likelihood function on the basis of the  $\widehat{\boldsymbol{\Sigma}}$  obtained from the underlying VAR. The short-run SVAR model approach chooses  $\mathbf{P} = \widehat{\mathbf{A}}^{-1} \widehat{\mathbf{B}}$  to identify the causal IRFs. The long-run SVAR model approach works similarly, with  $\mathbf{P} = \widehat{\mathbf{C}} = \widehat{\mathbf{A}}^{-1} \widehat{\mathbf{B}}$ , where  $\widehat{\mathbf{A}}^{-1}$  is the matrix of estimated long-run or accumulated effects of the reduced-form VAR model shocks.

There is one important difference between long-run and short-run SVAR models. As discussed by Amisano and Giannini (1997, chap. 6), in the short-run model the constraints are applied directly to the parameters in  $\mathbf{A}$  and  $\mathbf{B}$ . Then  $\mathbf{A}$  and  $\mathbf{B}$  interact with the estimated parameters of the underlying VAR model. In contrast, in a long-run model, the constraints are placed on functions of the estimated VAR model parameters. Although estimation and inference of the parameters in  $\mathbf{C}$  is straightforward, obtaining the asymptotic standard errors of the structural IRFs requires untenable assumptions. For this reason, `irf create` does not estimate the asymptotic standard errors of the structural IRFs generated by long-run SVAR models. However, bootstrap standard errors are still available.

In an instrumental-variables SVAR model, the SVAR model equations are set up as a short-run SVAR model with  $\mathbf{A} = \mathbf{I}_k$ ; thus,  $\mathbf{P} = \widehat{\mathbf{B}}$ . One or more columns of  $\widehat{\mathbf{B}}$  are estimated using an instrument to obtain identification. Structural IRFs are available for the shocks associated with the identified columns of  $\widehat{\mathbf{B}}$ .

## An introduction to dynamic-multiplier functions for VAR models

A dynamic-multiplier function measures the effect of a unit change in an exogenous variable on the endogenous variables over time. Per Lütkepohl (2005, chap. 10), if the VAR model with exogenous variables is stable, it can be rewritten as

$$\mathbf{y}_t = \sum_{i=0}^{\infty} \mathbf{D}_i x_{t-i} + \sum_{i=0}^{\infty} \boldsymbol{\Phi}_i u_{t-i}$$

where the  $\mathbf{D}_i$  are the dynamic-multiplier functions. (See *Methods and formulas* for details.) Some authors refer to the dynamic-multiplier functions as transfer functions because they specify how a unit change in an exogenous variable is “transferred” to the endogenous variables.

### □ Technical note

`irf create` computes dynamic-multiplier functions only after `var`. After short-run SVAR models, the dynamic multipliers from the VAR model are the same as those from the SVAR model. The dynamic multipliers for long-run SVAR models have not yet been worked out.

□

## An introduction to forecast-error variance decompositions for VAR models

Another measure of the effect of the innovations in variable  $k$  on variable  $j$  is the FEVD. This method, which is also known as *innovation accounting*, measures the fraction of the error in forecasting variable  $j$  after  $h$  periods that is attributable to the orthogonalized innovations in variable  $k$ . Because deriving the FEVD requires orthogonalizing the  $\mathbf{u}_t$  innovations, the FEVD is always predicated upon a choice of  $\mathbf{P}$ .

Lütkepohl (2005, sec. 2.2.2) shows that the  $h$ -step forecast error can be written as

$$\mathbf{y}_{t+h} - \widehat{\mathbf{y}}_t(h) = \sum_{i=0}^{h-1} \Phi_i \mathbf{u}_{t+h-i} \quad (4)$$

where  $y_{t+h}$  is the value observed at time  $t+h$  and  $\widehat{y}_t(h)$  is the  $h$ -step-ahead predicted value for  $y_{t+h}$  that was made at time  $t$ .

Because the  $\mathbf{u}_t$  are contemporaneously correlated, their distinct contributions to the forecast error cannot be ascertained. However, if we choose a  $\mathbf{P}$  such that  $\Sigma = \mathbf{P}\mathbf{P}'$ , as above, we can orthogonalize the  $\mathbf{u}_t$  into  $\mathbf{w}_t = \mathbf{P}^{-1}\mathbf{u}_t$ . We can then ascertain the relative contribution of the distinct elements of  $\mathbf{w}_t$ . Thus we can rewrite (4) as

$$\begin{aligned} \mathbf{y}_{t+h} - \widehat{\mathbf{y}}_t(h) &= \sum_{i=0}^{h-1} \Phi_i \mathbf{P}\mathbf{P}^{-1} \mathbf{u}_{t+h-i} \\ &= \sum_{i=0}^{h-1} \Theta_i \mathbf{w}_{t+h-i} \end{aligned}$$

Because the forecast errors can be written in terms of the orthogonalized errors, the forecast-error variance can be written in terms of the orthogonalized error variances. Forecast-error variance decompositions measure the fraction of the total forecast-error variance that is attributable to each orthogonalized shock.

### □ Technical note

The details in this note are not critical to the discussion that follows. A forecast-error variance decomposition is derived for a given  $\mathbf{P}$ . Per Lütkepohl (2005, sec. 2.3.3), letting  $\theta_{mn,i}$  be the  $m$ ,  $n$ th element of  $\Theta_i$ , we can express the  $h$ -step forecast error of the  $j$ th component of  $\mathbf{y}_t$  as

$$\begin{aligned} y_{j,t+h} - \widehat{y}_j(h) &= \sum_{i=0}^{h-1} \theta_{j1,1} \mathbf{w}_{1,t+h-i} + \cdots + \theta_{jK,i} \mathbf{w}_{K,t+h-i} \\ &= \sum_{k=1}^K \theta_{jk,0} \mathbf{w}_{k,t+h} + \cdots + \theta_{jk,h-1} \mathbf{w}_{k,t+1} \end{aligned}$$

The  $\mathbf{w}_t$ , which were constructed using  $\mathbf{P}$ , are mutually orthogonal with unit variance. This allows us to compute easily the mean squared error (MSE) of the forecast of variable  $j$  at horizon  $h$  in terms of the contributions of the components of  $\mathbf{w}_t$ . Specifically,

$$E[\{y_{j,t+h} - y_{j,t}(h)\}^2] = \sum_{k=1}^K (\theta_{jk,0}^2 + \cdots + \theta_{jk,h-1}^2)$$

The  $k$ th term in the sum above is interpreted as the contribution of the orthogonalized innovations in variable  $k$  to the  $h$ -step forecast error of variable  $j$ . Note that the  $k$ th element in the sum above can be rewritten as

$$(\theta_{jk,0}^2 + \cdots + \theta_{jk,h-1}^2) = \sum_{i=0}^{h-1} (\mathbf{e}'_j \boldsymbol{\Theta}_k \mathbf{e}_k)^2$$

where  $\mathbf{e}_i$  is the  $i$ th column of  $\mathbf{I}_K$ . Normalizing by the forecast error for variable  $j$  at horizon  $h$  yields

$$\omega_{jk,h} = \frac{\sum_{i=0}^{h-1} (\mathbf{e}'_j \boldsymbol{\Theta}_k \mathbf{e}_k)^2}{\text{MSE}\{y_{j,t}(h)\}}$$

where  $\text{MSE}\{y_{j,t}(h)\} = \sum_{i=0}^{h-1} \sum_{k=1}^K \theta_{jk,i}^2$ . □

Because the FEVD depends on the choice of  $\mathbf{P}$ , there are different forecast-error variance decompositions associated with each distinct  $\mathbf{P}$ . `irf create` can estimate the FEVD for a VAR model or an SVAR model. For a VAR model,  $\mathbf{P}$  is the Cholesky decomposition of  $\widehat{\boldsymbol{\Sigma}}$ . For an SVAR model,  $\mathbf{P}$  is the estimated structural decomposition,  $\mathbf{P} = \widehat{\mathbf{A}}^{-1} \widehat{\mathbf{B}}$  for short-run models and  $\mathbf{P} = \widehat{\mathbf{C}}$  for long-run SVAR models. Due to the same complications that arose with the structural impulse–response functions, the asymptotic standard errors of the structural FEVD are not available after long-run SVAR models, but bootstrap standard errors are still available.

## IRF results for VEC models

### An introduction to impulse–response functions for VEC models

As discussed in [TS] `vec intro`, the VEC model is a reparameterization of the VAR model that is especially useful for fitting VAR models with cointegrating variables. This implies that the estimated parameters for the corresponding VAR model can be backed out from the estimated parameters of the VEC model. This relationship means we can use the VAR form of the cointegrating VEC model to discuss the IRFs for VEC models.

Consider a cointegrating VAR model with one lag with no constant or trend,

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{u}_t \tag{5}$$

where  $\mathbf{y}_t$  is a  $K \times 1$  vector of endogenous, first-difference stationary variables among which there are  $1 \leq r < K$  cointegration equations;  $\mathbf{A}$  is  $K \times K$  matrix of parameters; and  $\mathbf{u}_t$  is a  $K \times 1$  vector of i.i.d. disturbances.

We developed intuition for the IRFs from a stationary VAR model by rewriting the VAR model as an infinite-order vector moving-average (VMA) process. While the Granger representation theorem establishes the existence of a VMA formulation of this model, because the cointegrating VAR model is not stable, the inversion is not nearly so intuitive. (See Johansen [1995, chapters 3 and 4] for more details.) For this reason, we use (5) to develop intuition for the IRFs from a cointegrating VAR model.

Suppose that  $K$  is 3, that  $\mathbf{u}_1 = (1, 0, 0)$ , and that we want to analyze the time paths of the variables in  $\mathbf{y}$  conditional on the initial values  $\mathbf{y}_0 = \mathbf{0}$ ,  $\mathbf{A}$ , and the condition that there are no more shocks to the system, that is,  $\mathbf{0} = \mathbf{u}_2 = \mathbf{u}_3 = \dots$ . These assumptions and (5) imply that

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{u}_1 \\ \mathbf{y}_2 &= \mathbf{A}\mathbf{y}_1 = \mathbf{A}\mathbf{u}_1 \\ \mathbf{y}_3 &= \mathbf{A}\mathbf{y}_2 = \mathbf{A}^2\mathbf{u}_1 \end{aligned}$$

and so on. The  $i$ th-row element of the first column of  $\mathbf{A}^s$  contains the effect of the unit shock to the first variable after  $s$  periods. The first column of  $\mathbf{A}^s$  contains the IRF of a unit impulse to the first variable after  $s$  periods. We could deduce the IRFs of a unit impulse to any of the other variables by administering the unit shock to one of them instead of to the first variable. Thus we can see that the  $(i, j)$ th element of  $\mathbf{A}^s$  contains the unit IRF from variable  $j$  to variable  $i$  after  $s$  periods. By starting with orthogonalized shocks of the form  $\mathbf{P}^{-1}\mathbf{u}_t$ , we can use the same logic to derive the OIRFs to be  $\mathbf{A}^s\mathbf{P}$ .

For the stationary VAR model, stability implies that all the eigenvalues of  $\mathbf{A}$  have moduli strictly less than one, which in turn implies that all the elements of  $\mathbf{A}^s \rightarrow \mathbf{0}$  as  $s \rightarrow \infty$ . This implies that all the IRFs from a stationary VAR model taper off to zero as  $s \rightarrow \infty$ . In contrast, in a cointegrating VAR model, some of the eigenvalues of  $\mathbf{A}$  are 1, while the remaining eigenvalues have moduli strictly less than 1. This implies that in cointegrating VAR models some of the elements of  $\mathbf{A}^s$  are not going to zero as  $s \rightarrow \infty$ , which in turn implies that some of the IRFs and OIRFs are not going to zero as  $s \rightarrow \infty$ . The fact that the IRFs and OIRFs taper off to zero for stationary VAR models but not for cointegrating VAR models is one of the key differences between the two models.

When the IRF or OIRF from the innovation in one variable to another tapers off to zero as time goes on, the innovation to the first variable is said to have a transitory effect on the second variable. When the IRF or OIRF does not go to zero, the effect is said to be permanent.

Note that, because some of the IRFs and OIRFs do not taper off to zero, some of the cumulative IRFs and OIRFs diverge over time.

## An introduction to forecast-error variance decompositions for VEC models

The results from *An introduction to impulse-response functions for VEC models* can be used to show that the interpretation of FEVDs for a finite number of steps in cointegrating VAR models is essentially the same as in the stationary case. Because the MSE of the forecast is diverging, this interpretation is valid only for a finite number of steps. (See [TS] [vec intro](#) and [TS] [fcst compute](#) for more information on this point.)

## IRF results for ARIMA and ARFIMA

A covariance-stationary additive ARMA( $p, q$ ) model can be written as

$$\rho(L^p)(y_t - \mathbf{x}_t\boldsymbol{\beta}) = \theta(L^q)\epsilon_t$$

where

$$\begin{aligned} \rho(L^p) &= 1 - \rho_1L - \rho_2L^2 - \dots - \rho_pL^p \\ \theta(L^q) &= 1 + \theta_1L + \theta_2L^2 + \dots + \theta_qL^q \end{aligned}$$

and  $L^j y_t = y_{t-j}$ .

We can rewrite the above model as an infinite-order moving-average process

$$y_t = \mathbf{x}_t\boldsymbol{\beta} + \boldsymbol{\psi}(L)\epsilon_t$$

where

$$\boldsymbol{\psi}(L) = \frac{\boldsymbol{\theta}(L)}{\boldsymbol{\rho}(L)} = 1 + \psi_1 L + \psi_2 L^2 + \dots \quad (6)$$

This representation shows the impact of the past innovations on the current  $y_t$ . The  $i$ th coefficient describes the response of  $y_t$  to a one-time impulse in  $\epsilon_{t-i}$ , holding everything else constant. The  $\psi_i$  coefficients are collectively referred to as the impulse–response function of the ARMA model. For a covariance-stationary series, the  $\psi_i$  coefficients decay exponentially.

A covariance-stationary multiplicative seasonal ARMA model, often abbreviated SARMA, of order  $(p, q) \times (P, Q)_s$  can be written as

$$\boldsymbol{\rho}(L^p)\boldsymbol{\rho}_s(L^P)(y_t - \mathbf{x}_t\boldsymbol{\beta}) = \boldsymbol{\theta}(L^q)\boldsymbol{\theta}_s(L^Q)\epsilon_t$$

where

$$\begin{aligned} \boldsymbol{\rho}_s(L^P) &= (1 - \rho_{s,1}L^s - \rho_{s,2}L^{2s} - \dots - \rho_{s,P}L^{Ps}) \\ \boldsymbol{\theta}_s(L^Q) &= (1 + \theta_{s,1}L^s + \theta_{s,2}L^{2s} + \dots + \theta_{s,Q}L^{Qs}) \end{aligned}$$

with  $\boldsymbol{\rho}(L^p)$  and  $\boldsymbol{\theta}(L^q)$  defined as above.

We can express this model as an additive ARMA model by multiplying the terms and imposing nonlinear constraints on multiplied coefficients. For example, consider the SARMA model given by

$$(1 - \rho_1 L)(1 - \rho_{4,1} L^4)y_t = \epsilon_t$$

Expanding the above equation and solving for  $y_t$  yields

$$y_t = \rho_1 y_{t-1} + \rho_{4,1} y_{t-4} - \rho_1 \rho_{4,1} y_{t-5} + \epsilon_t$$

or, in ARMA terms,

$$y_t = \rho_1 y_{t-1} + \rho_4 y_{t-4} + \rho_5 y_{t-5} + \epsilon_t$$

subject to the constraint  $\rho_5 = -\rho_1 \rho_{4,1}$ .

Once we have obtained an ARMA representation of a SARMA process, we obtain the IRFs from (6).

An ARFIMA( $p, d, q$ ) model can be written as

$$\boldsymbol{\rho}(L^p)(1 - L)^d(y_t - \mathbf{x}_t\boldsymbol{\beta}) = \boldsymbol{\theta}(L^q)\epsilon_t$$

with  $(1 - L)^d$  denoting a fractional integration operation.

Solving for  $y_t$ , we obtain

$$y_t = \mathbf{x}_t\boldsymbol{\beta} + (1 - L)^{-d}\boldsymbol{\psi}(L)\epsilon_t$$

This makes it clear that the impulse–response function for an ARFIMA model corresponds to a fractionally differenced impulse–response function for an ARIMA model. Because of the fractional differentiation, the  $\psi_i$  coefficients decay very slowly; see [Remarks and examples](#) in [TS] [arfima](#).

## Methods and formulas

Methods and formulas are presented under the following headings:

*Impulse–response function formulas for VAR models*  
*Dynamic-multiplier function formulas for VAR models*  
*Forecast-error variance decomposition formulas for VAR models*  
*Impulse–response function formulas for VEC models*  
*Algorithms for bootstrapping the VAR IRF and FEVD standard errors*  
*Impulse–response function formulas for ARIMA and ARFIMA*

### Impulse–response function formulas for VAR models

The previous discussion implies that there are three different choices of  $\mathbf{P}$  that can be used to obtain distinct  $\Theta_i$ .  $\mathbf{P}$  is the Cholesky decomposition of  $\Sigma$  for the OIRFs. For the structural IRFs,  $\mathbf{P} = \mathbf{A}^{-1}\mathbf{B}$  for short-run models, and  $\mathbf{P} = \mathbf{C}$  for long-run models. We will distinguish between the three by defining  $\Theta_i^o$  to be the OIRFs,  $\Theta_i^{\text{sr}}$  to be the short-run structural IRFs, and  $\Theta_i^{\text{lr}}$  to be the long-run structural IRFs.

We also define  $\widehat{\mathbf{P}}_c$  to be the Cholesky decomposition of  $\widehat{\Sigma}$ ,  $\widehat{\mathbf{P}}_{\text{sr}} = \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{B}}$  to be the short-run structural decomposition, and  $\widehat{\mathbf{P}}_{\text{lr}} = \widehat{\mathbf{C}}$  to be the long-run structural decomposition.

Given estimates of the  $\widehat{\mathbf{A}}_i$  and  $\widehat{\Sigma}$  from `var` or `svar`, the estimates of the simple IRFs and the OIRFs are, respectively,

$$\widehat{\Phi}_i = \sum_{j=1}^i \widehat{\Phi}_{i-j} \widehat{\mathbf{A}}_j$$

and

$$\widehat{\Theta}_i^o = \widehat{\Phi}_i \widehat{\mathbf{P}}_c$$

where  $\widehat{\mathbf{A}}_j = \mathbf{0}_K$  for  $j > p$ .

Given the estimates  $\widehat{\mathbf{A}}$  and  $\widehat{\mathbf{B}}$ , or  $\widehat{\mathbf{C}}$ , from `svar`, the estimates of the structural IRFs are either

$$\widehat{\Theta}_i^{\text{sr}} = \widehat{\Phi}_i \widehat{\mathbf{P}}_{\text{sr}}$$

or

$$\widehat{\Theta}_i^{\text{lr}} = \widehat{\Phi}_i \widehat{\mathbf{P}}_{\text{lr}}$$

The estimated structural IRFs stored in an IRF file with the variable name `sirf` may be from either a short-run model or a long-run model, depending on the estimation results used to create the IRFs. As discussed in [TS] [irf describe](#), you can easily determine whether the structural IRFs were generated from a short-run or a long-run SVAR model using `irf describe`.

Following [Lütkepohl \(2005, sec. 3.7\)](#), estimates of the cumulative IRFs and the cumulative orthogonalized impulse–response functions (COIRFs) at period  $n$  are, respectively,

$$\widehat{\Psi}_n = \sum_{i=0}^n \widehat{\Phi}_i$$

and

$$\widehat{\Xi}_n = \sum_{i=0}^n \widehat{\Theta}_i$$

The asymptotic standard errors of the different impulse–response functions are obtained by applications of the delta method. See Lütkepohl (2005, sec. 3.7) and Amisano and Giannini (1997, chap. 4) for the derivations. See Serfling (1980, sec. 3.3) for a discussion of the delta method. In presenting the variance–covariance matrix estimators, we make extensive use of the  $\text{vec}()$  operator, where  $\text{vec}(\mathbf{X})$  is the vector obtained by stacking the columns of  $\mathbf{X}$ .

Lütkepohl (2005, sec. 3.7) derives the asymptotic VCEs of  $\text{vec}(\Phi_i)$ ,  $\text{vec}(\Theta_i^o)$ ,  $\text{vec}(\hat{\Psi}_n)$ , and  $\text{vec}(\hat{\Xi}_n)$ . Because  $\text{vec}(\Phi_i)$  is  $K^2 \times 1$ , the asymptotic VCE of  $\text{vec}(\Phi_i)$  is  $K^2 \times K^2$ , and it is given by

$$\mathbf{G}_i \hat{\Sigma}_\alpha \mathbf{G}_i'$$

where

$$\begin{aligned} \mathbf{G}_i &= \sum_{m=0}^{i-1} \mathbf{J}(\widehat{\mathbf{M}}')^{(i-1-m)} \otimes \widehat{\Phi}_m & \mathbf{G}_i \text{ is } K^2 \times K^2 p \\ \mathbf{J} &= (\mathbf{I}_K, \mathbf{0}_K, \dots, \mathbf{0}_K) & \mathbf{J} \text{ is } K \times Kp \\ \widehat{\mathbf{M}} &= \begin{bmatrix} \widehat{\mathbf{A}}_1 & \widehat{\mathbf{A}}_2 & \dots & \widehat{\mathbf{A}}_{p-1} & \widehat{\mathbf{A}}_p \\ \mathbf{I}_K & \mathbf{0}_K & \dots & \mathbf{0}_K & \mathbf{0}_K \\ \mathbf{0}_K & \mathbf{I}_K & & \mathbf{0}_K & \mathbf{0}_K \\ \vdots & & \ddots & \vdots & \vdots \\ \mathbf{0}_K & \mathbf{0}_K & \dots & \mathbf{I}_K & \mathbf{0}_K \end{bmatrix} & \widehat{\mathbf{M}} \text{ is } Kp \times Kp \end{aligned}$$

The  $\widehat{\mathbf{A}}_i$  are the estimates of the coefficients on the lagged variables in the VAR model, and  $\widehat{\Sigma}_\alpha$  is the VCE matrix of  $\widehat{\alpha} = \text{vec}(\widehat{\mathbf{A}}_1, \dots, \widehat{\mathbf{A}}_p)$ .  $\widehat{\Sigma}_\alpha$  is a  $K^2 p \times K^2 p$  matrix whose elements come from the VCE of the VAR model coefficient estimator. As such, this VCE is the VCE of the constrained estimator if there are any constraints placed on the VAR model coefficients.

The  $K^2 \times K^2$  asymptotic VCE matrix for  $\text{vec}(\hat{\Psi}_n)$  after  $n$  periods is given by

$$\mathbf{F}_n \hat{\Sigma}_\alpha \mathbf{F}_n'$$

where

$$\mathbf{F}_n = \sum_{i=1}^n \mathbf{G}_i$$

The  $K^2 \times K^2$  asymptotic VCE matrix of the vectorized, orthogonalized, IRFs at horizon  $i$ ,  $\text{vec}(\Theta_i^o)$ , is

$$\mathbf{C}_i \hat{\Sigma}_\alpha \mathbf{C}_i' + \overline{\mathbf{C}}_i \hat{\Sigma}_\sigma \overline{\mathbf{C}}_i'$$

where

$$\begin{aligned}
 \mathbf{C}_0 &= \mathbf{0} & \mathbf{C}_0 \text{ is } K^2 \times K^2 p \\
 \mathbf{C}_i &= (\widehat{\mathbf{P}}'_c \otimes \mathbf{I}_K) \mathbf{G}_i, \quad i = 1, 2, \dots & \mathbf{C}_i \text{ is } K^2 \times K^2 p \\
 \overline{\mathbf{C}}_i &= (\mathbf{I}_K \otimes \Phi_i) \mathbf{H}, \quad i = 0, 1, \dots & \overline{\mathbf{C}}_i \text{ is } K^2 \times K^2 \\
 \mathbf{H} &= \mathbf{L}'_K \left\{ \mathbf{L}_K \mathbf{N}_K (\widehat{\mathbf{P}}_c \otimes \mathbf{I}_K) \mathbf{L}'_K \right\}^{-1} & \mathbf{H} \text{ is } K^2 \times K \frac{(K+1)}{2} \\
 \mathbf{L}_K \text{ solves } \quad \text{vech}(\mathbf{F}) &= \mathbf{L}_K \text{vec}(\mathbf{F}) & \mathbf{L}_K \text{ is } K \frac{(K+1)}{2} \times K^2 \\
 & \text{for } \mathbf{F} \text{ } K \times K \text{ and symmetric} \\
 \mathbf{K}_K \text{ solves } \quad \mathbf{K}_K \text{vec}(\mathbf{G}) &= \text{vec}(\mathbf{G}') \text{ for any } K \times K \text{ matrix } \mathbf{G} & \mathbf{K}_K \text{ is } K^2 \times K^2 \\
 \mathbf{N}_K &= \frac{1}{2} (\mathbf{I}_{K^2} + \mathbf{K}_K) & \mathbf{N}_K \text{ is } K^2 \times K^2 \\
 \widehat{\Sigma}_\sigma &= 2\mathbf{D}_K^+ (\widehat{\Sigma} \otimes \widehat{\Sigma}) \mathbf{D}_K^+ & \widehat{\Sigma}_\sigma \text{ is } K \frac{(K+1)}{2} \times K \frac{(K+1)}{2} \\
 \mathbf{D}_K^+ &= (\mathbf{D}'_K \mathbf{D}_K)^{-1} \mathbf{D}'_K & \mathbf{D}_K^+ \text{ is } K \frac{(K+1)}{2} \times K^2 \\
 \mathbf{D}_K \text{ solves } \quad \mathbf{D}_K \text{vech}(\mathbf{F}) &= \text{vec}(\mathbf{F}) \quad \text{for } \mathbf{F} \text{ } K \times K \text{ and symmetric} & \mathbf{D}_K \text{ is } K^2 \times K \frac{(K+1)}{2}
 \end{aligned}$$

$$\text{vech}(\mathbf{X}) = \begin{bmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{K1} \\ x_{22} \\ \vdots \\ x_{K2} \\ \vdots \\ x_{KK} \end{bmatrix} \quad \text{for } \mathbf{X} \text{ } K \times K \quad \text{vech}(\mathbf{X}) \text{ is } K \frac{(K+1)}{2} \times 1$$

Note that  $\widehat{\Sigma}_\sigma$  is the VCE of  $\text{vech}(\widehat{\Sigma})$ . More details about  $\mathbf{L}_K$ ,  $\mathbf{K}_K$ ,  $\mathbf{D}_K$  and  $\text{vech}()$  are available in [Lütkepohl \(2005, sec. A.12\)](#). Finally, as [Lütkepohl \(2005, 113–114\)](#) discusses,  $\mathbf{D}_K^+$  is the Moore–Penrose inverse of  $\mathbf{D}_K$ .

As discussed in [Amisano and Giannini \(1997, chap. 6\)](#), the asymptotic standard errors of the structural IRFs are available for short-run SVAR models but not for long-run SVAR models. Following [Amisano and Giannini \(1997, chap. 5\)](#), the asymptotic  $K^2 \times K^2$  VCE of the short-run structural IRFs after  $i$  periods, when a maximum of  $h$  periods are estimated, is the  $i, i$  block of

$$\widehat{\Sigma}(h)_{ij} = \widetilde{\mathbf{G}}_i \widehat{\Sigma}_\sigma \widetilde{\mathbf{G}}'_j + \left\{ \mathbf{I}_K \otimes (\mathbf{J} \widehat{\mathbf{M}}^i \mathbf{J}') \right\} \Sigma(0) \left\{ \mathbf{I}_K \otimes (\mathbf{J} \widehat{\mathbf{M}}^j \mathbf{J}') \right\}'$$

where

$$\begin{aligned}
 \tilde{\mathbf{G}}_0 &= \mathbf{0}_K & \mathbf{G}_0 & \text{is } K^2 \times K^2 p \\
 \tilde{\mathbf{G}}_i &= \sum_{k=0}^{i-1} \left\{ \widehat{\mathbf{P}}'_{sr} \mathbf{J} (\widehat{\mathbf{M}}')^{i-1-k} \otimes \left( \mathbf{J} \widehat{\mathbf{M}}^k \mathbf{J}' \right) \right\} & \mathbf{G}_i & \text{is } K^2 \times K^2 p \\
 \widehat{\Sigma}(0) &= \mathbf{Q}_2 \widehat{\Sigma}_W \mathbf{Q}'_2 & \widehat{\Sigma}(0) & \text{is } K^2 \times K^2 \\
 \widehat{\Sigma}_W &= \mathbf{Q}_1 \widehat{\Sigma}_{AB} \mathbf{Q}'_1 & \widehat{\Sigma}_W & \text{is } K^2 \times K^2 \\
 \mathbf{Q}_2 &= \widehat{\mathbf{P}}'_{sr} \otimes \widehat{\mathbf{P}}_{sr} & \mathbf{Q}_2 & \text{is } K^2 \times K^2 \\
 \mathbf{Q}_1 &= \left\{ (\mathbf{I}_K \otimes \widehat{\mathbf{B}}^{-1}), (-\widehat{\mathbf{P}}'^{-1}_{sr} \otimes \mathbf{B}^{-1}) \right\} & \mathbf{Q}_1 & \text{is } K^2 \times 2K^2
 \end{aligned}$$

and  $\widehat{\Sigma}_{AB}$  is the  $2K^2 \times 2K^2$  VCE of the estimator of  $\text{vec}(\mathbf{A}, \mathbf{B})$ .

Instrumental-variables SVAR models follow these formulas with  $\widehat{\mathbf{A}} = \mathbf{I}_k$  and elements of the identified columns of  $\widehat{\mathbf{B}}$  estimated through either GMM or minimum distance.

## Dynamic-multiplier function formulas for VAR models

This section provides the details of how `irf create` estimates the dynamic-multiplier functions and their asymptotic standard errors.

A  $p$ th order VAR model with exogenous variables may be written as

$$\mathbf{y}_t = \mathbf{v} + \mathbf{A}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \mathbf{B}_0 \mathbf{x}_t + \mathbf{B}_1 \mathbf{x}_{t-1} + \cdots + \mathbf{B}_s \mathbf{x}_{t-s} + \mathbf{u}_t$$

where all the notation is the same as above except that the  $s$   $K \times R$  matrices  $\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_s$  are explicitly included and  $s$  is the number of lags of the  $R$  exogenous variables in the model.

Lütkepohl (2005) shows that the dynamic-multipliers  $\mathbf{D}_i$  are consistently estimated by

$$\widehat{\mathbf{D}}_i = \mathbf{J}_x \widehat{\mathbf{A}}_x^i \widehat{\mathbf{B}}_x \quad i \in \{0, 1, \dots\}$$

where

$$\begin{aligned}
 \mathbf{J}_x &= (\mathbf{I}_K, \mathbf{0}_K, \dots, \mathbf{0}_K) & \mathbf{J} & \text{is } K \times (Kp + Rs) \\
 \widehat{\mathbf{A}}_x &= \begin{bmatrix} \widehat{\mathbf{M}} & \widehat{\mathbf{B}} \\ \mathbf{0} & \widehat{\mathbf{I}} \end{bmatrix} & \widehat{\mathbf{A}}_x & \text{is } (Kp + Rs) \times (Kp + Rs) \\
 \widehat{\mathbf{B}} &= \begin{bmatrix} \widehat{\mathbf{B}}_1 & \widehat{\mathbf{B}}_2 & \dots & \widehat{\mathbf{B}}_s \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix} & \widehat{\mathbf{B}} & \text{is } Kp \times Rs \\
 \widehat{\mathbf{I}} &= \begin{bmatrix} \mathbf{0}_R & \mathbf{0}_R & \dots & \mathbf{0}_R & \mathbf{0}_R \\ \mathbf{I}_R & \mathbf{0}_R & \dots & \mathbf{0}_R & \mathbf{0}_R \\ \mathbf{0}_R & \mathbf{I}_R & \dots & \mathbf{0}_R & \mathbf{0}_R \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0}_R & \mathbf{0}_R & \dots & \mathbf{I}_R & \mathbf{0}_R \end{bmatrix} & \widehat{\mathbf{I}} & \text{is } Rs \times Rs \\
 \widehat{\mathbf{B}}'_x &= [\widehat{\mathbf{B}}' \quad \widehat{\mathbf{I}}'] & \widehat{\mathbf{B}}'_x & \text{is } R \times (Kp + Rs) \\
 \widetilde{\mathbf{B}}' &= [\widetilde{\mathbf{B}}'_0 \quad \mathbf{0}' \dots \mathbf{0}'] & \widetilde{\mathbf{B}} & \text{is } R \times Kp \\
 \widetilde{\mathbf{I}}' &= [\mathbf{I}_R \quad \mathbf{0}_R \dots \mathbf{0}_R] & \widetilde{\mathbf{I}} & \text{is } R \times Rs
 \end{aligned}$$

and  $\mathbf{0}$  is a  $K \times R$  matrix of 0s and  $\widetilde{\mathbf{0}}$  is a  $Rs \times Kp$  matrix of 0s.

Consistent estimators of the cumulative dynamic-multiplier functions are given by

$$\bar{\mathbf{D}}_i = \sum_{j=0}^i \widehat{\mathbf{D}}_j$$

Letting  $\beta_x = \text{vec}(\mathbf{A}_1 \mathbf{A}_2 \cdots \mathbf{A}_p \mathbf{B}_1 \mathbf{B}_2 \cdots \mathbf{B}_s \mathbf{B}_0)$  and letting  $\Sigma_{\widehat{\beta}_x}$  be the asymptotic variance-covariance estimator (VCE) of  $\widehat{\beta}_x$ , Lütkepohl shows that an asymptotic VCE of  $\widehat{\mathbf{D}}_i$  is  $\widetilde{\mathbf{G}}_i \Sigma_{\widehat{\beta}_x} \widetilde{\mathbf{G}}_i'$  where

$$\widetilde{\mathbf{G}}_i = \left( \sum_{j=0}^{i-1} \mathbf{B}'_x \widetilde{\mathbf{A}}_x^{i-1-j} \otimes \mathbf{J}_x \widetilde{\mathbf{A}}_x^j \mathbf{J}'_x, \mathbf{I}_R \otimes \mathbf{J}_x \widetilde{\mathbf{A}}_x^j \mathbf{J}_x \right)$$

Similarly, an asymptotic VCE of  $\bar{\mathbf{D}}_i$  is  $\left( \sum_{j=0}^i \widetilde{\mathbf{G}}_j \right) \Sigma_{\widehat{\beta}_x} \left( \sum_{j=0}^i \widetilde{\mathbf{G}}_j' \right)$ .

## Forecast-error variance decomposition formulas for VAR models

This section provides details of how `irf create` estimates the Cholesky FEVD, the structural FEVD, and their standard errors. Beginning with the Cholesky-based forecast-error decompositions, the fraction of the  $h$ -step-ahead forecast-error variance of variable  $j$  that is attributable to the Cholesky orthogonalized innovations in variable  $k$  can be estimated as

$$\widehat{\omega}_{jk,h} = \frac{\sum_{i=0}^{h-1} (\mathbf{e}'_j \widehat{\Theta}_i \mathbf{e}_k)^2}{\widehat{\text{MSE}}_j(h)}$$

where  $\text{MSE}_j(h)$  is the  $j$ th diagonal element of

$$\sum_{i=0}^{h-1} \widehat{\Phi}_i \widehat{\Sigma} \widehat{\Phi}_i'$$

(See Lütkepohl [2005, 109] for a discussion of this result.)  $\widehat{\omega}_{jk,h}$  and  $\text{MSE}_j(h)$  are scalars. The square of the standard error of  $\widehat{\omega}_{jk,h}$  is

$$\mathbf{d}_{jk,h} \widehat{\Sigma}_\alpha \mathbf{d}'_{jk,h} + \bar{\mathbf{d}}_{jk,h} \widehat{\Sigma}_\sigma \bar{\mathbf{d}}_{jk,h}$$

where

$$\begin{aligned} \mathbf{d}_{jk,h} &= \frac{2}{\widehat{\text{MSE}}_j(h)^2} \sum_{i=0}^{h-1} \left\{ \text{MSE}_j(h) (\mathbf{e}'_j \widehat{\Phi}_i \widehat{\mathbf{P}}_c \mathbf{e}_k) (\mathbf{e}'_k \widehat{\mathbf{P}}'_c \otimes \mathbf{e}'_j) \mathbf{G}_i \right. \\ &\quad \left. - (\mathbf{e}'_j \widehat{\Phi}_i \widehat{\mathbf{P}}_c \mathbf{e}_k)^2 \sum_{m=0}^{h-1} (\mathbf{e}'_j \widehat{\Phi}_m \widehat{\Sigma} \otimes \mathbf{e}'_j) \mathbf{G}_m \right\} \quad \mathbf{d}_{jk,h} \text{ is } 1 \times K^2 p \\ \bar{\mathbf{d}}_{jk,h} &= \sum_{i=0}^{h-1} \left\{ \text{MSE}_j(h) (\mathbf{e}'_j \widehat{\Phi}_i \widehat{\mathbf{P}}_c \mathbf{e}_k) (\mathbf{e}'_k \otimes \mathbf{e}'_j \widehat{\Phi}_i) \mathbf{H} \right. \\ &\quad \left. - (\mathbf{e}'_j \widehat{\Phi}_i \widehat{\mathbf{P}}_c \mathbf{e}_k)^2 \sum_{m=0}^{h-1} (\mathbf{e}'_j \widehat{\Phi}_m \otimes \mathbf{e}_j \widehat{\Phi}_m) \mathbf{D}_K \right\} \frac{1}{\widehat{\text{MSE}}_j(h)^2} \quad \bar{\mathbf{d}}_{jk,h} \text{ is } 1 \times K \frac{(K+1)}{2} \\ \mathbf{G}_0 &= \mathbf{0} \quad \mathbf{G}_0 \text{ is } K^2 \times K^2 p \end{aligned}$$

and  $\mathbf{D}_K$  is the  $K^2 \times K\{(K+1)/2\}$  duplication matrix defined previously.

For the structural forecast-error decompositions, we follow [Amisano and Giannini \(1997, sec. 5.2\)](#). They define the matrix of structural forecast-error decompositions at horizon  $s$ , when a maximum of  $h$  periods are estimated, as

$$\begin{aligned}\widehat{\mathbf{W}}_s &= \widehat{\mathbf{F}}_s^{-1} \widehat{\mathbf{M}}_s \quad \text{for } s = 1, \dots, h+1 \\ \widehat{\mathbf{F}}_s &= \left( \sum_{i=0}^{s-1} \widehat{\boldsymbol{\Theta}}_i^{\text{SR}} \widehat{\boldsymbol{\Theta}}_i^{\text{SR}'} \right) \odot \mathbf{I}_K \\ \widehat{\mathbf{M}}_s &= \sum_{i=0}^{s-1} \widehat{\boldsymbol{\Theta}}_i^{\text{SR}} \odot \widehat{\boldsymbol{\Theta}}_i^{\text{SR}}\end{aligned}$$

where  $\odot$  is the Hadamard, or element-by-element, product.

The  $K^2 \times K^2$  asymptotic VCE of  $\text{vec}(\widehat{\mathbf{W}}_s)$  is given by

$$\widetilde{\mathbf{Z}}_s \boldsymbol{\Sigma}(h) \widetilde{\mathbf{Z}}_s'$$

where  $\widehat{\boldsymbol{\Sigma}}(h)$  is as derived previously, and

$$\begin{aligned}\widetilde{\mathbf{Z}}_s &= \left\{ \frac{\partial \text{vec}(\widehat{\mathbf{W}}_s)}{\partial \text{vec}(\widehat{\boldsymbol{\Theta}}_0^{\text{SR}})}, \frac{\partial \text{vec}(\widehat{\mathbf{W}}_s)}{\partial \text{vec}(\widehat{\boldsymbol{\Theta}}_1^{\text{SR}})}, \dots, \frac{\partial \text{vec}(\widehat{\mathbf{W}}_s)}{\partial \text{vec}(\widehat{\boldsymbol{\Theta}}_h^{\text{SR}})} \right\} \\ \frac{\partial \text{vec}(\widehat{\mathbf{W}}_s)}{\partial \text{vec}(\widehat{\boldsymbol{\Theta}}_j^{\text{SR}})} &= 2 \left\{ (\mathbf{I}_K \otimes \widehat{\mathbf{F}}_s^{-1}) \widetilde{\mathbf{D}}(\widehat{\boldsymbol{\Theta}}_j^{\text{SR}}) - (\widehat{\mathbf{W}}_s' \otimes \widehat{\mathbf{F}}_s^{-1}) \widetilde{\mathbf{D}}(\mathbf{I}_K) \mathbf{N}_K (\widehat{\boldsymbol{\Theta}}_j^{\text{SR}} \otimes \mathbf{I}_K) \right\}\end{aligned}$$

If  $\mathbf{X}$  is an  $n \times n$  matrix, then  $\widetilde{\mathbf{D}}(\mathbf{X})$  is the  $n^2 \times n^2$  matrix with  $\text{vec}(\mathbf{X})$  on the diagonal and zeros in all the off-diagonal elements, and  $\mathbf{N}_K$  is as defined previously.

## Impulse–response function formulas for VEC models

We begin by providing the formulas for backing out the estimates of the  $\mathbf{A}_i$  from the  $\boldsymbol{\Gamma}_i$  estimated by  $\text{vec}$ . As discussed in [\[TS\] vec intro](#), the VAR model in [\(1\)](#) can be rewritten as a VEC model:

$$\Delta y_t = v + \boldsymbol{\Pi} y_{t-1} + \boldsymbol{\Gamma}_1 \Delta y_{t-1} + \boldsymbol{\Gamma}_{p-1} \Delta y_{p-2} + \epsilon_t$$

$\text{vec}$  estimates  $\boldsymbol{\Pi}$  and the  $\boldsymbol{\Gamma}_i$ . [Johansen \(1995, 25\)](#) notes that

$$\boldsymbol{\Pi} = \sum_{i=1}^p \mathbf{A}_i - \mathbf{I}_K \quad (6)$$

where  $\mathbf{I}_K$  is the  $K$ -dimensional identity matrix, and

$$\boldsymbol{\Gamma}_i = - \sum_{j=i+1}^p \mathbf{A}_j \quad (7)$$

Defining

$$\mathbf{\Gamma} = \mathbf{I}_K - \sum_{i=1}^{p-1} \mathbf{\Gamma}_i$$

and using (6) and (7) allow us to solve for the  $\mathbf{A}_i$  as

$$\mathbf{A}_1 = \mathbf{\Pi} + \mathbf{\Gamma}_1 + \mathbf{I}_K$$

$$\mathbf{A}_i = \mathbf{\Gamma}_i - \mathbf{\Gamma}_{i-1} \quad \text{for } i = \{2, \dots, p-1\}$$

and

$$\mathbf{A}_p = -\mathbf{\Gamma}_{p-1}$$

Using these formulas, we can back out estimates of  $\mathbf{A}_i$  from the estimates of the  $\mathbf{\Gamma}_i$  and  $\mathbf{\Pi}$  produced by `vec`. Then we simply use the formulas for the IRFs and OIRFs presented in [Impulse–response function formulas for VAR models](#).

The running sums of the IRFs and OIRFs over the steps within each impulse–response pair are the cumulative IRFs and OIRFs.

## Algorithms for bootstrapping the VAR IRF and FEVD standard errors

`irf create` offers two bootstrap algorithms for estimating the standard errors of the various IRFs and FEVDs. Both `var` and `svar` contain estimators for the coefficients in a VAR model that are conditional on the first  $p$  observations. The two bootstrap algorithms are also conditional on the first  $p$  observations.

Specifying the `bs` option calculates the standard errors by bootstrapping the residuals. For a bootstrap with  $R$  repetitions, this method uses the following algorithm:

1. Fit the model and save the estimated parameters.
2. Use the estimated coefficients to calculate the residuals.
3. Repeat steps 3a to 3c  $R$  times.
  - 3a. Draw a simple random sample of size  $T$  with replacement from the residuals. The random samples are drawn over the  $K \times 1$  vectors of residuals. When the  $t$ th vector is drawn, all  $K$  residuals are selected. This preserves the contemporaneous correlations among the residuals.
  - 3b. Use the  $p$  initial observations, the sampled residuals, and the estimated coefficients to construct a new sample dataset.
  - 3c. Fit the model and calculate the different IRFs and FEVDs.
  - 3d. Save these estimates as observation  $r$  in the bootstrapped dataset.
4. For each IRF and FEVD, the estimated standard deviation from the  $R$  bootstrapped estimates is the estimated standard error of that impulse–response function or forecast-error variance decomposition.

Specifying the `bsp` option estimates the standard errors by a multivariate normal parametric bootstrap. The algorithm for the multivariate normal parametric bootstrap is identical to the one above, with the exception that 3a is replaced by 3a(bsp):

- 3a(bsp). Draw  $T$  pseudovariates from a multivariate normal distribution with covariance matrix  $\hat{\mathbf{\Sigma}}$ .

## Impulse–response function formulas for ARIMA and ARFIMA

The previous discussion showed that a SARMA process can be rewritten as an ARMA process and that for an ARMA process, we can express  $\psi(L)$  in terms of  $\theta(L)$  and  $\rho(L)$ ,

$$\psi(L) = \frac{\theta(L)}{\rho(L)}$$

Expanding the above, we obtain

$$\psi_0 + \psi_1 L + \psi_2 L^2 + \dots = \frac{1 + \theta_1 L + \theta_2 L^2 + \dots}{1 - \rho_1 L - \rho_2 L^2 - \dots}$$

Given the estimate of the autoregressive terms  $\hat{\rho}$  and the moving-average terms  $\hat{\theta}$ , the IRF is obtained by solving the above equation for the  $\psi$  weights. The  $\psi_i$  are calculated using the recursion

$$\hat{\psi}_i = \hat{\theta}_i + \sum_{j=1}^p \hat{\phi}_j \hat{\psi}_{i-j}$$

with  $\psi_0 = 1$  and  $\theta_i = 0$  for  $i > \max(p, q + 1)$ .

The asymptotic standard errors for the IRF for ARMA are calculated using the delta method; see [Serfling \(1980, sec. 3.3\)](#) for a discussion of the delta method. Let  $\hat{\Sigma}$  be the estimate of the variance–covariance matrix for  $\hat{\rho}$  and  $\hat{\theta}$ , and let  $\Psi$  be a matrix of derivatives of  $\psi_i$  with respect to  $\hat{\rho}$  and  $\hat{\theta}$ . Then the standard errors for  $\hat{\psi}_i$  are calculated as

$$\Psi_i \hat{\Sigma} \Psi_i'$$

The IRF for the ARFIMA( $p, d, q$ ) model is obtained by applying the filter  $(1 - L)^{-d}$  to  $\psi(L)$ . The filter is given by [Hassler and Kokoszka \(2010\)](#) as

$$(1 - L)^{-d} = \sum_{i=0}^{\infty} b_i L^i$$

with  $b_0 = 1$  and subsequent  $b_i$  calculated by the recursion

$$\hat{b}_i = \frac{\hat{d} + i - 1}{i} \hat{b}_{i-1}$$

The resulting IRF is then given by

$$\hat{\phi}_i = \sum_{j=0}^i \hat{\psi}_j \hat{b}_{i-j}$$

The asymptotic standard errors for the IRF for ARFIMA are calculated using the delta method. Let  $\hat{\Sigma}$  be the estimate of the variance–covariance matrix for  $\hat{\rho}$ ,  $\hat{\theta}$ , and  $\hat{d}$ , and let  $\Phi$  be a matrix of derivatives of  $\phi_i$  with respect to  $\hat{\rho}$ ,  $\hat{\theta}$ , and  $\hat{d}$ . Then the standard errors for  $\hat{\phi}_i$  are calculated as

$$\Phi_i \hat{\Sigma} \Phi_i'$$

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## Also see

- [TS] **irf** — Create and analyze IRFs, dynamic-multiplier functions, and FEVDs
- [TS] **var intro** — Introduction to vector autoregressive models
- [TS] **vec intro** — Introduction to vector error-correction models

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