Example 31g — Two-factor measurement model (generalized response)

Description Remarks and examples Also see

Description

We demonstrate a two-factor generalized linear measurement model with the same data used in [SEM] **Example 29g**:

```
. use https://www.stata-press.com/data/r18/gsem_cfa
(Fictional math abilities data)
. describe
Contains data from https://www.stata-press.com/data/r18/gsem_cfa.dta
 Observations:           500                  Fictional math abilities data
    Variables:            19                  21 Mar 2022 10:38
                                              (_dta has notes)
```

Variable name	Storage type	Display format	Value label	Variable label
school	byte	%9.0g		School ID
id	long	%9.0g		Student ID
q1	byte	%9.0g	result	q1 correct
q2	byte	%9.0g	result	q2 correct
q3	byte	%9.0g	result	q3 correct
q4	byte	%9.0g	result	q4 correct
q5	byte	%9.0g	result	q5 correct
q6	byte	%9.0g	result	q6 correct
q7	byte	%9.0g	result	q7 correct
q8	byte	%9.0g	result	q8 correct
att1	byte	%26.0g	agree	Skills taught in math class will help me get a better job.
att2	byte	%26.0g	agree	Math is important in everyday life
att3	byte	%26.0g	agree	Working math problems makes me anxious.
att4	byte	%26.0g	agree	Math has always been my worst subject.
att5	byte	%26.0g	agree	I am able to learn new math concepts easily.
test1	byte	%9.0g		Score, math test 1
test2	byte	%9.0g		Score, math test 2
test3	byte	%9.0g		Score, math test 3
test4	byte	%9.0g		Score, math test 4

```
Sorted by:
. notes
_dta:
  1.  Fictional data on math ability and attitudes of 500 students from 20
      schools.
  2.  Variables q1-q8 are incorrect/correct (0/1) on individual math questions.
  3.  Variables att1-att5 are items from a Likert scale measuring each
      student's attitude toward math.
  4.  Variables test1-test4 are test scores from tests of four different
      aspects of mathematical abilities.  Range of scores: 0-100.
```

These data record results from a fictional instrument measuring mathematical ability. Variables `q1` through `q8` are the items from the instrument.

In this example, we will also be using variables `att1` through `att5`. These are five Likert-scale questions measuring each student's attitude toward math.

See *Multiple-factor measurement models* in [SEM] **Intro 5** for background.

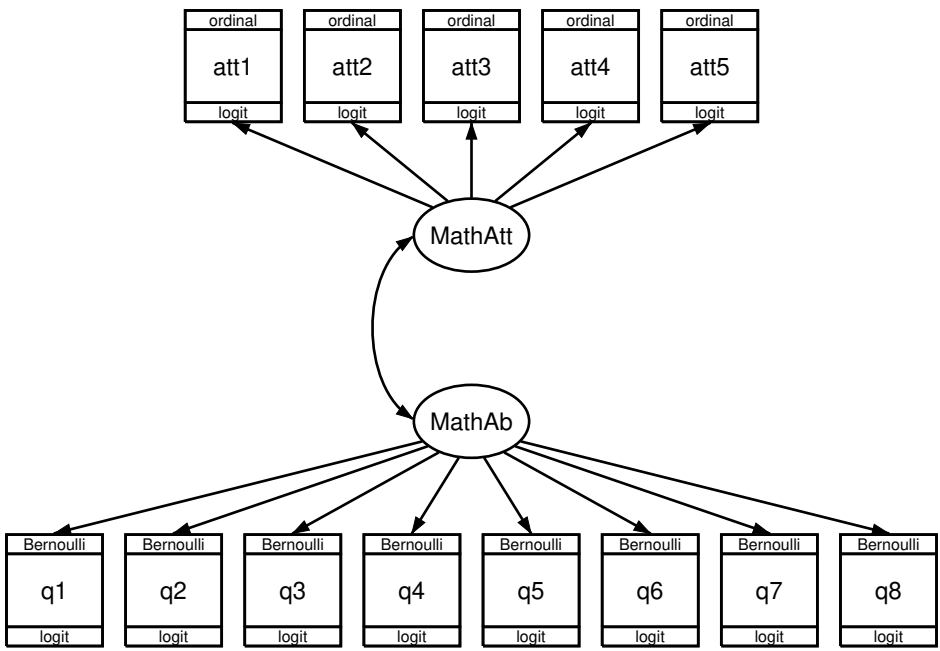
Remarks and examples

stata.com

Remarks are presented under the following headings:

Fitting the two-factor model

We extend the measurement model fit in [SEM] **Example 29g** from one factor, math ability, to two factors, math ability and attitude. The model we wish to fit is



In this model, mathematical ability affects the correctness of the answers to the items just as previously. The new component, attitude toward mathematics, is correlated with math ability. We expect this correlation to be positive, but that is yet to be determined.

What is important about the attitudinal questions is that the responses are ordinal, that is, the ordering of the possible answers is significant. In other cases, we might have a categorical variable taking on, say, five values; even if the values are 1, 2, 3, 4, and 5, there is no case in which answer 5 is greater than answer 4, answer 4 is greater than answer 3, and so on.

For our attitude measures, however, response 5 signifies strong agreement with a statement and 1 signifies strong disagreement. We handle the ordinal property by specifying that the attitudinal responses are family ordinal, link logit, also known as ordered logit or ordinal logistic regression, and also known in Stata circles as ologit.

In the command language, to fit a one-factor measurement model with math ability, we would type

```
gsem (MathAb -> q1-q8), logit
```

To include the second factor, attitude correlated with math ability, we would type

```
gsem (MathAb  -> q1-q8,      logit)     ///
     (MathAtt -> att1-att5, ologit)
```

The covariance between `MathAtt` and `MathAb` does not even appear in the command! That is because latent exogenous variables are assumed to be correlated in the command language unless you specify otherwise; in path diagrams, such variables are correlated only if a curved path is drawn between them.

There is another, minor difference in syntax between the one-factor and two-factor models that is worth your attention. Notice that the `logit` was outside the parentheses in the command to fit the one-factor model, but it is inside the parentheses in the command to fit the two-factor model. Actually, `logit` could have appeared inside the parentheses to fit the one-factor model. When options appear inside parentheses, they affect only what is specified inside the parentheses. When they appear outside parentheses, they affect all parenthetical specifications.

To obtain the estimates of the two-factor model, we type

```
. gsem (MathAb -> q1-q8, logit)
>      (MathAtt -> att1-att5, ologit)

Fitting fixed-effects model:

Iteration 0:  Log likelihood = -6629.7253
Iteration 1:  Log likelihood = -6628.7848
Iteration 2:  Log likelihood = -6628.7848

Refining starting values:

Grid node 0:  Log likelihood = -6457.4584

Fitting full model:

Iteration 0:  Log likelihood = -6457.4584
Iteration 1:  Log likelihood = -6437.9594
Iteration 2:  Log likelihood = -6400.2731
Iteration 3:  Log likelihood = -6396.3795
Iteration 4:  Log likelihood = -6394.5787
Iteration 5:  Log likelihood = -6394.4019
Iteration 6:  Log likelihood = -6394.3923
Iteration 7:  Log likelihood = -6394.3923

Generalized structural equation model                     Number of obs = 500

Response: q1
Family:   Bernoulli
Link:     Logit

Response: q2
Family:   Bernoulli
Link:     Logit

Response: q3
Family:   Bernoulli
Link:     Logit

Response: q4
Family:   Bernoulli
Link:     Logit
```

```
Response: q5
Family:   Bernoulli
Link:     Logit

Response: q6
Family:   Bernoulli
Link:     Logit

Response: q7
Family:   Bernoulli
Link:     Logit

Response: q8
Family:   Bernoulli
Link:     Logit

Response: att1
Family:   Ordinal
Link:     Logit

Response: att2
Family:   Ordinal
Link:     Logit

Response: att3
Family:   Ordinal
Link:     Logit

Response: att4
Family:   Ordinal
Link:     Logit

Response: att5
Family:   Ordinal
Link:     Logit

Log likelihood = -6394.3923

 ( 1)  [q1]MathAb = 1
 ( 2)  [att1]MathAtt = 1
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
q1						
MathAb	1	(constrained)				
_cons	.0446118	.1272964	0.35	0.726	-.2048845	.2941082
q2						
MathAb	.3446081	.1050264	3.28	0.001	.1387601	.5504562
_cons	-.4572215	.0979965	-4.67	0.000	-.6492911	-.265152
q3						
MathAb	.5445245	.1386993	3.93	0.000	.272679	.8163701
_cons	.1591406	.1033116	1.54	0.123	-.0433464	.3616276
q4						
MathAb	.2858874	.0948553	3.01	0.003	.0999743	.4718004
_cons	-.3196648	.0947684	-3.37	0.001	-.5054075	-.1339222
q5						
MathAb	.8174803	.1867024	4.38	0.000	.4515504	1.18341
_cons	-.04543	.116575	-0.39	0.697	-.2739127	.1830527
q6						
MathAb	.6030448	.1471951	4.10	0.000	.3145478	.8915419
_cons	-.309992	.1070853	-2.89	0.004	-.5198753	-.1001086

q7						
MathAb	.72084	.1713095	4.21	0.000	.3850796	1.056601
_cons	.1047265	.1116494	0.94	0.348	-.1141023	.3235552
q8						
MathAb	.5814761	.1426727	4.08	0.000	.3018428	.8611094
_cons	-.0250442	.1045134	-0.24	0.811	-.2298868	.1797983
att1						
MathAtt	1	(constrained)				
att2						
MathAtt	.3788714	.0971223	3.90	0.000	.1885152	.5692276
att3						
MathAtt	-1.592717	.3614859	-4.41	0.000	-2.301216	-.8842173
att4						
MathAtt	-.8100107	.153064	-5.29	0.000	-1.11001	-.5100108
att5						
MathAtt	.5225423	.1170141	4.47	0.000	.2931988	.7518858
/att1						
cut1	-1.10254	.1312272			-1.359741	-.8453396
cut2	-.2495339	.1160385			-.4769651	-.0221027
cut3	.2983261	.1164414			.0701052	.5265471
cut4	1.333053	.1391907			1.060244	1.605861
/att2						
cut1	-1.055791	.1062977			-1.264131	-.8474513
cut2	-.1941211	.0941435			-.378639	-.0096032
cut3	.3598488	.0952038			.1732528	.5464448
cut4	1.132624	.1082204			.9205156	1.344732
/att3						
cut1	-1.053519	.1733999			-1.393377	-.7136614
cut2	-.0491073	.1442846			-.3318999	.2336853
cut3	.5570671	.1538702			.2554871	.8586471
cut4	1.666859	.2135554			1.248298	2.08542
/att4						
cut1	-1.07378	.1214071			-1.311734	-.8358264
cut2	-.2112462	.1076501			-.4222366	-.0002559
cut3	.406347	.1094847			.191761	.620933
cut4	1.398185	.1313327			1.140778	1.655593
/att5						
cut1	-1.244051	.1148443			-1.469142	-1.018961
cut2	-.336135	.0986678			-.5295203	-.1427498
cut3	.2137776	.0978943			.0219084	.4056468
cut4	.9286849	.107172			.7186316	1.138738
var(MathAb)	2.300652	.7479513			1.216527	4.350909
var(MathAtt)	1.520854	.4077674			.8992196	2.572228
cov(MathAb, MathAtt)	.8837681	.2204606	4.01	0.000	.4516733	1.315863

Notes:

1. The estimated covariance between math attitude and ability is 0.88.
2. There is something new in the output, namely, things labeled `cut1`, ..., `cut4`. These appear for each of the five attitudinal measures. These are the ordered logit's cutpoints, the values on the logit's distribution that separate attitude 1 from attitude 2, attitude 2 from attitude 3, and so on. The four cutpoints map the continuous distribution into five ordered, categorical groups.
3. There's something interesting hiding in the `MathAtt` coefficients: the coefficients for two of the paths, `att3 att4 <- MathAtt`, are negative! If you look back to the description of the data, you will find that the sense of these two questions was reversed from those of the other questions. Strong agreement on these two questions was agreement with a negative feeling about mathematics.

`estat sd` displays the fitted variance components as standard deviations and correlations. From the following, we see that the estimated correlation between attitude and ability is 0.4725.

```
. estat sd
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
sd(MathAb)	1.51679	.2465573			1.102963	2.085883
sd(MathAtt)	1.233229	.1653251			.9482719	1.603817
corr(MathAb, MathAtt)	.4724644	.0649541	7.27	0.000	.3357955	.5896506

Fitting the model with the Builder

Use the diagram in *Fitting the two-factor model* above for reference.

1. Open the dataset.

   In the Command window, type

   ```
   . use https://www.stata-press.com/data/r18/gsem_cfa
   ```

2. Open a new Builder diagram.

   Select menu item **Statistics > SEM (structural equation modeling) > Model building and estimation**.

3. Put the Builder in `gsem` mode by clicking on the G SEM button.

4. Create the measurement component for `MathAb`.

   Select the Add measurement component tool, , and then click in the diagram about one-third of the way up from the bottom and slightly left of the center.

   In the resulting dialog box,

   a. change the *Latent variable name* to `MathAb`;

   b. select `q1`, `q2`, `q3`, `q4`, `q5`, `q6`, `q7`, and `q8` by using the *Measurement variables* control;

   c. check *Make measurements generalized*;

   d. select `Bernoulli, Logit` in the *Family/Link* control;

   e. select `Down` in the *Measurement direction* control;

   f. click on **OK**.

   If you wish, move the component by clicking on any variable and dragging it.

5. Create the measurement component for `MathAtt`.

   Select the Add measurement component tool, , and then click in the diagram about one-third of the way down from the top and slightly left of the center.

   In the resulting dialog box,

   a. change the *Latent variable name* to `MathAtt`;

   b. select `att1`, `att2`, `att3`, `att4`, and `att5` by using the *Measurement variables* control;

   c. check *Make measurements generalized*;

   d. select `Ordinal, Logit` in the *Family/Link* control;

   e. select `Up` in the *Measurement direction* control;

   f. click on **OK**.

   If you wish, move the component by clicking on any variable and dragging it.

6. Create the covariance between `MathAtt` and `MathAb`.

   a. Select the Add covariance tool, .

   b. Click in the top-left quadrant of the `MathAb` oval, and drag a covariance to the bottom left of the `MathAtt` oval.

7. Clean up.

   If you do not like where a covariance has been connected to its variable, use the Select tool, , to simply click on the covariance, and then click on where it connects to an oval and drag the endpoint. You can also change the bow of the covariance by dragging the control point that extends from one end of the selected covariance.

8. Estimate.

   Click on the **Estimate** button, , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

You can open a completed diagram in the Builder by typing

```
. webgetsem gsem_2fmm
```

Also see

[SEM] **Example 27g** — Single-factor measurement model (generalized response)

[SEM] **Example 29g** — Two-parameter logistic IRT model

[SEM] **Example 32g** — Full structural equation model (generalized response)

[SEM] **Intro 5** — Tour of models

[SEM] **gsem** — Generalized structural equation model estimation command

[SEM] **estat sd** — Display variance components as standard deviations and correlations