# Title

> **bitest** — Binomial probability test

## Description

bitest performs exact hypothesis tests for binomial random variables. The null hypothesis is that the probability of a success on a trial is $\#_p$. The total number of trials is the number of nonmissing values of *varname* (in bitest) or $\#_N$ (in bitesti). The number of observed successes is the number of 1s in *varname* (in bitest) or $\#_{\text{succ}}$ (in bitesti). *varname* must contain only 0s, 1s, and missing.

bitesti is the immediate form of bitest; see [U] **19 Immediate commands** for a general introduction to immediate commands.

## Quick start

Exact test for probability of success ($a = 1$) is 0.4

    bitest a = .4

With additional exact probabilities

    bitest a = .4, detail

Exact test that the probability of success is 0.46, given 22 successes in 74 trials

    bitesti 74 22 .46

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### bitesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Binomial probability test calculator

## Syntax

*Binomial probability test*

> bitest *varname*== #$_p$ [ *if* ] [ *in* ] [ *weight* ] [ , <u>detail</u> ]

*Immediate form of binomial probability test*

> bitesti #$_N$ #$_{succ}$ #$_p$ [ , <u>detail</u> ]

by and collect are allowed with bitest; see [U] **11.1.10 Prefix commands**.
fweights are allowed with bitest; see [U] **11.1.6 weight**.

## Option

> ┌─────────┐
> │ Advanced │
> └─────────┘
>
> <u>detail</u> shows the probability of the observed number of successes, $k_{obs}$; the probability of the number of successes on the opposite tail of the distribution that is used to compute the two-sided *p*-value, $k_{opp}$; and the probability of the point next to $k_{opp}$. This information can be safely ignored. See the technical note below for details.

## Remarks and examples

Remarks are presented under the following headings:

> bitest
> bitesti

### bitest

▷ Example 1

We test 15 university students for high levels of one measure of visual quickness which, from other evidence, we believe is present in 30% of the nonuniversity population. Included in our data is quick, taking on the values 1 ("success") or 0 ("failure") depending on the outcome of the test.

```
. use https://www.stata-press.com/data/r18/quick
. bitest quick == 0.3
Binomial probability test
```

| Variable | N | Observed k | Expected k | Assumed p | Observed p |
|---|---|---|---|---|---|
| quick | 15 | 7 | 4.5 | 0.30000 | 0.46667 |

```
Pr(k >= 7)           = 0.131143  (one-sided test)
Pr(k <= 7)           = 0.949987  (one-sided test)
Pr(k <= 1 or k >= 7) = 0.166410  (two-sided test)
```

The first part of the output reveals that, assuming a true probability of success of 0.3, the expected number of successes is 4.5 and that we observed seven. Said differently, the assumed frequency under the null hypothesis $H_0$ is 0.3, and the observed frequency is 0.47.

The first line under the table is a one-sided test; it is the probability of observing seven or more successes conditional on $p = 0.3$. It is a test of $H_0: p = 0.3$ versus the alternative hypothesis $H_A: p > 0.3$. Said in English, the alternative hypothesis is that more than 30% of university students score at high levels on this test of visual quickness. The $p$-value for this hypothesis test is 0.13.

The second line under the table is a one-sided test of $H_0$ versus the opposite alternative hypothesis $H_A: p < 0.3$.

The third line is the two-sided test. It is a test of $H_0$ versus the alternative hypothesis $H_A: p \neq 0.3$.

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## ❑ Technical note

The $p$-value of a hypothesis test is the probability (calculated assuming $H_0$ is true) of observing any outcome as extreme or more extreme than the observed outcome, with extreme meaning in the direction of the alternative hypothesis. In example 1, the outcomes $k = 8, 9, \ldots, 15$ are clearly "more extreme" than the observed outcome $k_{obs} = 7$ when considering the alternative hypothesis $H_A: p \neq 0.3$. However, outcomes with only a few successes are also in the direction of this alternative hypothesis. For two-sided hypotheses, outcomes with $k$ successes are considered "as extreme or more extreme" than the observed outcome $k_{obs}$ if $\Pr(k) \leq \Pr(k_{obs})$. Here $\Pr(k = 0)$ and $\Pr(k = 1)$ are both less than $\Pr(k = 7)$, so they are included in the two-sided $p$-value.

The `detail` option allows you to see the probability (assuming that $H_0$ is true) of the observed successes ($k = 7$) and the probability of the boundary point ($k = 1$) of the opposite tail used for the two-sided $p$-value.

```
. bitest quick == 0.3, detail

Binomial probability test
```

| Variable | N | Observed k | Expected k | Assumed p | Observed p |
|---|---|---|---|---|---|
| quick | 15 | 7 | 4.5 | 0.30000 | 0.46667 |

```
Pr(k >= 7)            =  0.131143  (one-sided test)
Pr(k <= 7)            =  0.949987  (one-sided test)
Pr(k <= 1 or k >= 7)  =  0.166410  (two-sided test)

Pr(k == 7)            =  0.081130  (observed)
Pr(k == 2)            =  0.091560
Pr(k == 1)            =  0.030520  (opposite extreme)
```

Also shown is the probability of the point next to the boundary point. This probability, namely, $\Pr(k = 2) = 0.092$, is certainly close to the probability of the observed outcome $\Pr(k = 7) = 0.081$, so some people might argue that $k = 2$ should be included in the two-sided $p$-value. Statisticians (at least some we know) would reply that the $p$-value is a precisely defined concept and that this is an arbitrary "fuzzification" of its definition. When you compute exact $p$-values according to the precise definition of a $p$-value, your type I error is never more than what you say it is—so no one can criticize you for being anticonservative. Including the point $k = 2$ is being overly conservative because it makes the $p$-value larger yet. But it is your choice; being overly conservative, at least in statistics, is always safe. Know that `bitest` and `bitesti` always keep to the precise definition of a $p$-value, so if you wish to include this extra point, you must do so by hand or by using the `r()` stored results; see *Stored results* below.

❑

## bitesti

▷ Example 2

The binomial test is a function of two statistics and one parameter: $N$, the number of observations; $k_{\text{obs}}$, the number of observed successes; and $p$, the assumed probability of a success on a trial. For instance, in a city of $N = 2{,}500{,}000$, we observe $k_{\text{obs}} = 36$ cases of a particular disease when the population rate for the disease is $p = 0.00001$.

```
. bitesti 2500000 36 .00001

Binomial probability test

              N   Observed k   Expected k   Assumed p   Observed p

       2,500,000          36           25     0.00001      0.00001

  Pr(k >= 36)              = 0.022458  (one-sided test)
  Pr(k <= 36)              = 0.985448  (one-sided test)
  Pr(k <= 14 or k >= 36)   = 0.034859  (two-sided test)
```

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## Stored results

bitest and bitesti store the following in r():

Scalars
| | |
|---|---|
| r(N) | number $N$ of trials |
| r(P_p) | assumed probability $p$ of success |
| r(k) | observed number $k$ of successes |
| r(p_l) | lower one-sided $p$-value |
| r(p_u) | upper one-sided $p$-value |
| r(p) | two-sided $p$-value |
| r(k_opp) | opposite extreme $k$ |
| r(P_k) | probability of observed $k$ (detail only) |
| r(P_oppk) | probability of opposite extreme $k$ (detail only) |
| r(k_nopp) | $k$ next to opposite extreme (detail only) |
| r(P_noppk) | probability of $k$ next to opposite extreme (detail only) |

## Methods and formulas

Let $N$, $k_{\text{obs}}$, and $p$ be, respectively, the number of observations, the observed number of successes, and the assumed probability of success on a trial. The expected number of successes is $Np$, and the observed probability of success on a trial is $k_{\text{obs}}/N$.

`bitest` and `bitesti` compute exact $p$-values based on the binomial distribution. The upper one-sided $p$-value is

$$\Pr(k \geq k_{\text{obs}}) = \sum_{m=k_{\text{obs}}}^{N} \binom{N}{m} p^m (1-p)^{N-m}$$

The lower one-sided $p$-value is

$$\Pr(k \leq k_{\text{obs}}) = \sum_{m=0}^{k_{\text{obs}}} \binom{N}{m} p^m (1-p)^{N-m}$$

If $k_{\text{obs}} \geq Np$, the two-sided $p$-value is

$$\Pr(k \leq k_{\text{opp}} \text{ or } k \geq k_{\text{obs}})$$

where $k_{\text{opp}}$ is the largest number $\leq Np$ such that $\Pr(k = k_{\text{opp}}) \leq \Pr(k = k_{\text{obs}})$. If $k_{\text{obs}} < Np$, the two-sided $p$-value is

$$\Pr(k \leq k_{\text{obs}} \text{ or } k \geq k_{\text{opp}})$$

where $k_{\text{opp}}$ is the smallest number $\geq Np$ such that $\Pr(k = k_{\text{opp}}) \leq \Pr(k = k_{\text{obs}})$.

## Reference

Hoel, P. G. 1984. *Introduction to Mathematical Statistics.* 5th ed. New York: Wiley.

## Also see

[R] **ci** — Confidence intervals for means, proportions, and variances

[R] **prtest** — Tests of proportions