> **meta regress postestimation** — Postestimation tools for meta regress

Postestimation commands

The following postestimation command is of special interest after `meta regress`:

Command	Description
estat bubbleplot	bubble plots

The following standard postestimation commands are also available:

Command	Description
contrast	contrasts and ANOVA-style joint tests of estimates
estat summarize	summary statistics for the estimation sample
estat vce	variance–covariance matrix of the estimators (VCE)
estimates	cataloging estimation results
etable	table of estimation results
lincom	point estimates, standard errors, testing, and inference for linear combinations of coefficients
margins	marginal means, predictive margins, marginal effects, and average marginal effects
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
nlcom	point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predict	predictions and their SEs, leverage statistics, etc.
predictnl	point estimates, standard errors, testing, and inference for generalized predictions
pwcompare	pairwise comparisons of estimates
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

predict

Description for predict

predict creates a new variable containing predictions such as linear predictions, residuals, leverage, and standard errors. After random-effects meta-regression, you can also obtain estimates of random effects and their standard errors.

Menu for predict

Statistics > Postestimation

Syntax for predict

Syntax for obtaining predictions other than best linear unbiased predictions (BLUPs) of random effects

> predict [*type*] *newvar* [*if*] [*in*] [, *statistic* <u>fixed</u>only se(*sespec*)]

Syntax for obtaining BLUPs of random effects and their standard errors after random-effects meta-regression

> predict [*type*] *newvar* [*if*] [*in*], <u>ref</u>fects [se(*newvar*) reses(*resesspec*)]

statistic	Description
Main	
xb	linear prediction; the default
stdp	standard error of the linear prediction
<u>fitted</u>	fitted values, fixed-portion linear prediction plus predicted random effects
<u>residuals</u>	residuals, response minus fitted values
<u>leverage</u>│hat	leverage (diagonal elements of hat matrix)

Unstarred statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

Options for predict

> ┌─── Main ┐

xb, the default, calculates the linear prediction $\mathbf{x}_j\widehat{\boldsymbol{\beta}}$. For the random-effects meta-regression, this corresponds to the fixed portion of the linear predictor based on the estimated regression coefficients. That is, this is equivalent to fixing all random effects in the model to their theoretical mean value of 0.

stdp calculates the standard error of the linear prediction.

fitted calculates the fitted values. With fixed-effects meta-regression or with random-effects meta-regression when option fixedonly is also specified, this option is equivalent to xb. For random-effects meta-regression without fixedonly, it calculates $\mathbf{x}_j\widehat{\boldsymbol{\beta}} + u_j$, which is equal to the fixed portion of the linear prediction plus predicted random effects.

residuals calculates the residuals, which are equal to the responses minus the fitted values. With fixed-effects meta-regression or with random-effects meta-regression when option fixedonly is also specified, it calculates $\widehat{\theta}_j - \mathbf{x}_j\widehat{\boldsymbol{\beta}}$. The former are known as marginal residuals in the context of the random-effects model. For random-effects meta-regression without fixedonly, this option calculates $\widehat{\theta}_j - (\mathbf{x}_j\widehat{\boldsymbol{\beta}} + u_j)$, which are known as conditional residuals.

leverage or hat calculates the diagonal elements of the projection ("hat") matrix.

fixedonly specifies that all random effects be set to zero, which is equivalent to using only the fixed portion of the model, when computing results for random-effects models. This option may be specified only with statistics fitted, residuals, or leverage.

reffects calculates best linear unbiased predictions (BLUPs) of the random effects.

se(*newvar* [, marginal]) calculates the standard errors of the corresponding predicted values. This option may be specified only with statistics reffects, fitted, and residuals. When specified with reffects, se(*newvar*) is a synonym to reses(*newvar*, diagnostic).

Suboption marginal is allowed only with random-effects meta-regression and requires option fixedonly. It computes marginal standard errors, when you type

. predict ..., *statistic* se(*newvar*, marginal) fixedonly

instead of the standard errors conditional on zero random effects, which are computed when you type

. predict ..., *statistic* se(*newvar*) fixedonly

marginal is not allowed in combination with reffects.

reses(*resesspec*) calculates the standard errors of the random effects; see option reffects. This option may not be combined with option se(). The syntax for *resesspec* is

   newvar [, comparative | diagnostic]

comparative, the default, computes comparative random-effects standard errors. For linear models, these correspond to posterior standard deviations of random effects and to standard errors of marginal prediction errors $\widehat{u}_j - u_j$. These standard errors are used for inference about the random effects.

diagnostic computes diagnostic random-effects standard errors. These correspond to marginal standard errors of BLUPs, $\mathrm{SE}(\widehat{u}_j)$. These standard errors are used for model diagnostics.

margins

Description for margins

margins estimates margins of response for linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

> margins [*marginlist*] [, *options*]
>
> margins [*marginlist*] , p̲redict(*statistic* ...) [*options*]

statistic	Description
xb	linear prediction; the default
fi̲tted	fitted values; implies fixedonly
stdp	not allowed with margins
res̲iduals	not allowed with margins
lev̲erage│hat	not allowed with margins
ref̲fects	not allowed with margins

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] **[margins](#)**.

Remarks and examples

We demonstrate some of the postestimation features, including estat bubbleplot, margins, and predict after meta regress.

▷ Example 1: Bubble plot

Consider the declared BCG dataset of clinical trials that studied the efficacy of a Bacillus Calmette-Guérin (BCG) vaccine in the prevention of tuberculosis (TB) (Colditz et al. 1994). In example 1 of [META] **meta regress**, we used meta regress to fit a simple meta-regression to these data with the continuous moderator latitude_c to explore heterogeneity.

```
. use https://www.stata-press.com/data/r18/bcgset
(Efficacy of BCG vaccine against tuberculosis; set with -meta esize-)

. meta regress latitude_c

  Effect-size label: Log risk-ratio
        Effect size: _meta_es
          Std. err.: _meta_se

Random-effects meta-regression                   Number of obs  =        13
Method: REML                                     Residual heterogeneity:
                                                             tau2 =   .07635
                                                           I2 (%) =    68.39
                                                               H2 =     3.16
                                                    R-squared (%) =    75.63
                                                    Wald chi2(1)  =    16.36
                                                    Prob > chi2   =   0.0001
```

_meta_es	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
latitude_c	-.0291017	.0071953	-4.04	0.000	-.0432043	-.0149991
_cons	-.7223204	.1076535	-6.71	0.000	-.9333174	-.5113234

```
Test of residual homogeneity: Q_res = chi2(11) = 30.73   Prob > Q_res = 0.0012
```

Whenever there is one continuous moderator in a meta-regression, a so-called bubble plot is commonly used to explore the relationship between the effect size and that moderator. Let's use estat bubbleplot to produce the bubble plot after the fitted meta-regression.

```
. estat bubbleplot
```


Weights: Inverse-variance
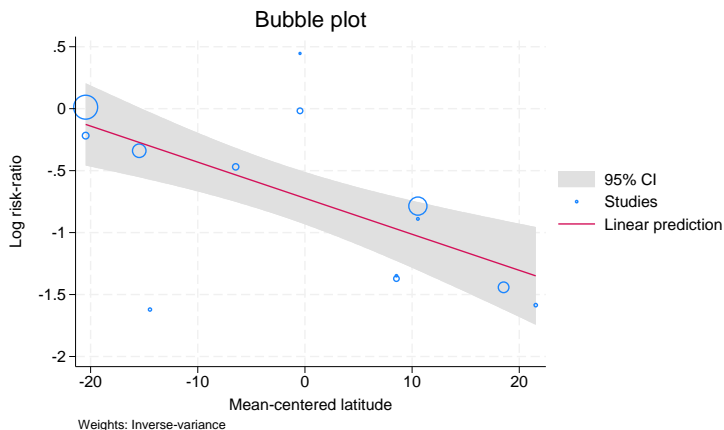
A bubble plot is a scatterplot of the observed effect sizes against the moderator overlaid with the predicted regression and confidence-intervals lines. Each study is represented by a circle (bubble) with the size (area) proportional to the study precision, $1/\widehat{\sigma}_j^2$. The larger the size of the bubble, the more precise the study. The coordinates of the center of each circle show the observed value of the effect size on the y axis and that of the moderator (latitude_c in our example) on the x axis. The solid line shows the predicted values (predicted log risk-ratios in our example). The predicted 95% confidence intervals are also plotted.

From the plot, the log risk-ratio for the BCG vaccine declines as the distance from the equator increases. There appear to be a couple of outlying studies (see points in the bottom left and middle top sections of the plot), but their bubbles are very small, which suggests that their log risk-ratios estimates had small weights, relative to other studies, in the meta-regression. Outlying studies with large bubbles may be a source of concern because of the large differences in their effect sizes compared with those from the other studies and because of the large impact they have on the regression results.

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▷ Example 2: Marginal effects

Continuing with example 1, we found that the log risk-ratio for the BCG decreases as the distance from the equator increases. For example, from the bubble plot, a trial conducted relatively close to the equator, say, in Thailand (with a latitude of 15 or a centered latitude of -18.5), would have a predicted log risk-ratio of about -0.2. A trial conducted in, say, Nepal (with a latitude of 28 or a centered latitude of -5.5), would have a predicted log risk-ratio of about -0.7. And a trial conducted in, say, Ukraine (with a latitude of 50 or a centered latitude of 16.5), would have a predicted log risk-ratio of about -1.

Instead of relying on the graph, we can obtain more precise estimates of the predicted log risk-ratios at different latitude values by using the margins command as follows:

```
. margins, at(latitude_c = (-18.5 -5.5 16.5))
Adjusted predictions                                    Number of obs = 13
Expression: Fitted values; fixed portion (xb), predict(fitted fixedonly)
1._at: latitude_c = -18.5
2._at: latitude_c =  -5.5
3._at: latitude_c =  16.5
```

	Margin	Delta-method std. err.	z	P>|z|	[95% conf. interval]
_at					
1	-.1839386	.1586092	-1.16	0.246	-.4948069 .1269297
2	-.562261	.1091839	-5.15	0.000	-.7762574 -.3482645
3	-1.202499	.1714274	-7.01	0.000	-1.53849 -.8665072

The list of numbers specified in the at() option are the values of the latitudes centered around the latitude mean (≈ 33.5).

Note that results produced by margins are on the log scale and need to be exponentiated to make interpretations on the natural (risk) scale. For instance, from the output, the risk ratio for regions with latitude_c = 16.5 is $\exp(-1.202499) = 0.3$, which means that the vaccine is expected to reduce the risk of TB by 70% for regions with that latitude.

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▷ Example 3: Predicted random effects

In example 1, we noticed a couple of outlying studies. Let's explore this further by looking at predicted random effects from our random-effects meta-regression.

We first use predict with options reffects and se() to predict the random-effects and estimate their diagnostic standard errors.

```
. predict double u, reffects se(se_u)
```

Then, we generate a new variable, ustandard, as the ratio of the predicted random effects to their diagnostic standard errors and use the qnorm command (see [R] **Diagnostic plots**) to construct the normal quantile plot.

```
. generate double ustandard = u/se_u
. label variable ustandard "Standardized predicted random effects"
. qnorm ustandard, mlabel(trial)
```
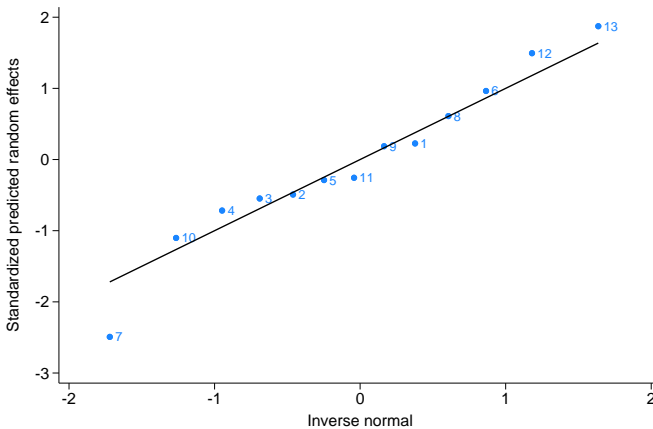

The plot suggests that trial 7, labeled "Vandiviere et al., 1973" in our data, is an outlier. From the data, the log risk-ratio estimate for this trial is -1.62 with the corresponding risk-ratio estimate of about 0.2. This means that, in that trial, the vaccine reduced the risk of TB by roughly 80% even though this trial was conducted relatively close to the equator (in Haiti, with latitude=19). In fact, this trial reported the largest risk reduction (smallest log-risk-ratio value) in the meta-analysis. Compare this with trial 11 ("Comstock et al., 1974"), which was conducted in Puerto Rico and has a similar latitude (latitude=18) but whose estimated risk reduction was much more moderate, about 29% (with the risk-ratio estimate of $\exp(-0.34) = 0.71$). More investigation is needed to explain the extreme value reported by trial 7. Thus, in this example, you may consider reporting the results of meta-analyses with and without this trial.

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Methods and formulas

Methods and formulas are presented under the following headings:

Random-effects meta-regression
Fixed-effects meta-regression

The following formulas are used by predict. The notation is based on *Methods and formulas* of [META] **meta regress**.

Random-effects meta-regression

The fixed-portion of the linear prediction (option xb) is $\mathbf{x}_j\widehat{\boldsymbol{\beta}}$. The estimated standard error of the fixed-portion of the linear prediction (option stdp) for study j is

$$\widehat{\mathrm{SE}}\left(\mathbf{x}_j\widehat{\boldsymbol{\beta}}\right) = \sqrt{\mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}'_j}$$

The BLUP of the jth random effect (option reffects) is

$$\widehat{u}_j = \lambda_j\left(\widehat{\theta}_j - \mathbf{x}_j\widehat{\boldsymbol{\beta}}\right)$$

where

$$\lambda_j = \frac{\widehat{\tau}^2}{\widehat{\tau}^2 + \widehat{\sigma}_j^2}$$

is the empirical Bayes shrinkage factor for the jth study. When the reses() option is also specified, the estimated comparative standard error of $\widehat{u}_j$ is

$$\widehat{\mathrm{SE}}\left(\widehat{u}_j - u_j\right) = \sqrt{\widehat{\tau}^2 - \lambda_j^2\left\{\widehat{\sigma}_j^2 + \widehat{\tau}^2 - \mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}'_j\right\}}$$

When suboption diagnostic of reses() is specified or when the se() option is specified, the estimated diagnostic standard error of $\widehat{u}_j$ is

$$\widehat{\mathrm{SE}}\left(\widehat{u}_j\right) = \lambda_j\sqrt{\widehat{\sigma}_j^2 + \widehat{\tau}^2 - \mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}'_j}$$

See Goldstein (2011), Skrondal and Rabe-Hesketh (2009), and Rabe-Hesketh and Skrondal (2022) for more details.

The fitted value (option fitted) is

$$\widetilde{\theta}_j = \mathbf{x}_j\widehat{\boldsymbol{\beta}} + \widehat{u}_j$$

When the se() option is also specified, the estimated standard error of $\widetilde{\theta}_j$ is

$$\widehat{\mathrm{SE}}\left(\widetilde{\theta}_j\right) = \sqrt{\lambda_j^2\left(\widehat{\sigma}_j^2 + \widehat{\tau}^2\right) + \left(1 - \lambda_j^2\right)\mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}'_j}$$

The residual (option residuals) is

$$e_j = \widehat{\theta}_j - \widetilde{\theta}_j$$

When option se() is also specified, the estimated standard error of e_j is

$$\widehat{\mathrm{SE}}\left(e_j\right) = \sqrt{\left(1 + \lambda_j^2\right)\left(\widehat{\sigma}_j^2 + \widehat{\tau}^2 - \mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}'_j\right)}$$

The leverage (option `hat`) are the diagonal elements of the hat matrix $\mathbf{X}\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{W}^*$:

$$h_j^* = \frac{1}{\widehat{\tau}^2 + \widehat{\sigma}_j^2}\mathbf{x}_j\left(\mathbf{X}'\mathbf{W}^*\mathbf{X}\right)^{-1}\mathbf{x}_j'$$

When the `fixedonly` option is specified, the formulas for the fitted values and residuals (including their standard errors) and leverage are adjusted by replacing the value of $\widehat{u}_j$ with 0, in which case, $\widehat{\tau}^2 = 0$, $\lambda_j = 0$, and $\mathbf{W}^*$ is replaced with $\mathbf{W} = \text{diag}\left(1/\widehat{\sigma}_1^2, \ldots, 1/\widehat{\sigma}_K^2\right)$. In this case, the standard errors are computed conditionally on zero random effects.

If `se()`'s option `marginal` is specified, then marginal standard errors are computed. This is equivalent to computing $\widehat{\text{SE}}(\widetilde{\theta}_j)$ and $\widehat{\text{SE}}\left(e_j\right)$ with $\lambda_j = 0$ but keeping $\widehat{\tau}^2$ and $\mathbf{W}^*$ unchanged.

Fixed-effects meta-regression

The linear prediction (option `xb`) is $\mathbf{x}_j\widehat{\boldsymbol{\beta}}$. The estimated standard error of the linear prediction (option `stdp`) for study j is

$$\widehat{\text{SE}}\left(\mathbf{x}_j\widehat{\boldsymbol{\beta}}\right) = \sqrt{\mathbf{x}_j\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{x}_j'}$$

The fitted value (option `fitted`) is the same as the linear prediction:

$$\widetilde{\theta}_j = \mathbf{x}_j\widehat{\boldsymbol{\beta}}$$

The residual (option `residuals`) is

$$e_j = \widehat{\theta}_j - \widetilde{\theta}_j$$

When option `se()` is also specified, the estimated standard error of e_j is

$$\widehat{\text{SE}}\left(e_j\right) = \sqrt{\left(\widehat{\sigma}_j^2 - \mathbf{x}_j\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{x}_j'\right)}$$

The leverage (option `hat`) are the diagonal elements of the hat matrix $\mathbf{X}\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{W}$:

$$h_j = \frac{1}{\widehat{\sigma}_j^2}\mathbf{x}_j\left(\mathbf{X}'\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{x}_j'$$

For the multiplicative fixed-effects meta-regression, in the above formulas, replace $\mathbf{W}$ with $\mathbf{W}^{\phi}$ and $\widehat{\sigma}_j^2$ with $\widehat{\phi}\widehat{\sigma}_j^2$, where $\widehat{\phi}$ is defined in *Fixed-effects meta-regression* in [META] **meta regress**.

References

Colditz, G. A., T. F. Brewer, C. S. Berkey, M. E. Wilson, E. Burdick, H. V. Fineberg, and F. Mosteller. 1994. Efficacy of BCG vaccine in the prevention of tuberculosis: Meta-analysis of the published literature. *Journal of the American Medical Association* 271: 698–702. https://doi.org/10.1001/jama.1994.03510330076038.

Goldstein, H. 2011. *Multilevel Statistical Models*. 4th ed. Chichester, UK: Wiley.

Rabe-Hesketh, S., and A. Skrondal. 2022. *Multilevel and Longitudinal Modeling Using Stata*. 4th ed. College Station, TX: Stata Press.

Skrondal, A., and S. Rabe-Hesketh. 2009. Prediction in multilevel generalized linear models. *Journal of the Royal Statistical Society, Series A* 172: 659–687. https://doi.org/10.1111/j.1467-985X.2009.00587.x.

Also see

[META] **meta regress** — Meta-analysis regression

[META] **meta data** — Declare meta-analysis data

[META] **meta** — Introduction to meta

[META] **Glossary**

[META] **Intro** — Introduction to meta-analysis

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