Title

> **estat heterogeneity** — Compute multivariate heterogeneity statistics

Description

estat heterogeneity computes multivariate heterogeneity statistics after a random-effects multivariate meta-regression fit by meta mvregress. By default, the multivariate Cochran and Jackson–White–Riley heterogeneity statistics are computed, but the White statistic is also available.

Menu for estat

Statistics > Postestimation

Syntax

estat <u>heter</u>ogeneity [, *statistics*]

statistics	Description
Main	
<u>cochran</u>	Cochran statistics; the default
<u>jwr</u>iley[(*depnames*)]	Jackson–White–Riley statistics; the default for all dependent variables
white	White statistics
all	all heterogeneity statistics

You may specify one or multiple statistics simultaneously.

collect is allowed; see [U] **11.1.10 Prefix commands**.

Options

> **Main**

cochran, the default, specifies that the multivariate Cochran I_Q^2 and H_Q^2 heterogeneity statistics be computed. These statistics are computed jointly for all dependent variables and thus are not available for each dependent variable separately. These statistics are displayed by default.

jwriley, the default, and jwriley(*depnames*) specify that Jackson–White–Riley heterogeneity statistics I_{JWR}^2 and R_{JWR} be computed. These statistics are available jointly for all dependent variables, separately for each dependent variable, or jointly for any subset of dependent variables. The statistics for all dependent variables, separately and jointly, are displayed by default.

jwriley computes the Jackson–White–Riley heterogeneity statistics jointly for all dependent variables and separately for each dependent variable. For example, if you have three dependent variables y1, y2, and y3, then the Jackson–White–Riley heterogeneity statistics are computed separately for each variable and jointly for all variables, resulting in four I^2_{JWR} statistics and four R_{JWR} statistics.

jwriley(*depnames*) computes the Jackson–White–Riley statistics jointly for the specified dependent variables *depnames*. For example, if you have three dependent variables y1, y2, and y3, you may specify jwriley(y1 y2) to compute heterogeneity statistics based on dependent variables y1 and y2 jointly.

white specifies that White I^2 statistics be computed. These statistics are available only separately for each dependent variable. With one dependent variable, the White I^2 statistic reduces to the univariate I^2 statistic as reported by [META] **meta summarize** or [META] **meta regress**.

all specifies that all heterogeneity statistics be reported. This option implies cochran, jwriley, and white.

Remarks and examples

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▷ Example 1: Jackson–White–Riley heterogeneity statistics

Consider the dataset from example 8 of [META] **meta mvregress**. We fit a random-effects multivariate meta-regression using maximum likelihood estimation and an exchangeable covariance structure for the between-study covariance matrix:

```
. use https://www.stata-press.com/data/r17/smokecess
(Smoking cessation interventions)
. quietly meta mvregress y*, wcovvariables(v*)
> random(mle, covariance(exchangeable))
```

We can then compute the multivariate Jackson–White–Riley heterogeneity statistics by typing

```
. estat heterogeneity, jwriley
Method: Jackson-White-Riley
yb:
  I2 (%) = 81.83
      R =  2.35
yc:
  I2 (%) = 90.60
      R =  3.26
yd:
  I2 (%) = 62.84
      R =  1.64
Joint:
  I2 (%) = 78.61
      R =  2.16
```

The jwriley option displays the Jackson–White–Riley heterogeneity statistics separately for each dependent variable and jointly for all dependent variables. See example 4 of [META] **meta mvregress** for the interpretation of these statistics.

One feature that is unique to the Jackson–White–Riley heterogeneity statistics is the possibility of assessing heterogeneity jointly for a subset of dependent variables. Below, we specify the jwriley(yb yc) option to assess heterogeneity jointly for these two dependent variables.

```
. estat heterogeneity, jwriley(yb yc)
Method: Jackson-White-Riley
yb yc:
  I2 (%) = 86.02
       R =  2.67
```

The value of I^2 is 86.02%, which suggests that there is considerable heterogeneity among the effect sizes corresponding to these variables; see example 4 of [META] **meta mvregress** for details about the interpretation of I^2_{JWR} and R_{JWR}.

◁

Stored results

estat heterogeneity stores the following in r():

Scalars
r(I2_Q)	Cochran I^2 heterogeneity statistic (option cochran)
r(H2)	H^2 heterogeneity statistic (option cochran)

Matrices
r(R)	Jackson–White–Riley R heterogeneity statistics (option jwriley)
r(I2_JWR)	Jackson–White–Riley I^2 heterogeneity statistics (option jwriley)
r(I2_W)	White I^2 heterogeneity statistics (option white)

Methods and formulas

Methods and formulas are presented under the following headings:

> Brief overview of heterogeneity statistics
> Cochran heterogeneity statistics
> Jackson–White–Riley heterogeneity statistics
> White heterogeneity statistics

See *Methods and formulas* of [META] **meta mvregress**.

Consider a random-effects multivariate meta-regression

$$\widehat{\boldsymbol{\theta}}_j = \mathbf{X}_j \boldsymbol{\beta} + \mathbf{u}_j + \boldsymbol{\epsilon}_j, \quad \boldsymbol{\epsilon}_j \sim N_d\left(\mathbf{0}, \boldsymbol{\Lambda}_j\right)$$

where $\mathbf{u}_j = (u_{1j}, u_{2j}, \ldots, u_{dj})' \sim N_d(\mathbf{0}, \boldsymbol{\Sigma})$ $(j = 1, \ldots, K)$ define random effects, $\mathbf{x}_j = (1, x_{1j}, \ldots, x_{p-1,j})$ is a $1 \times p$ vector of moderators (covariates), $\mathbf{X}_j = \mathbf{x}_j \otimes I_d$ is a $d \times dp$ design matrix, and $\boldsymbol{\beta} = (\boldsymbol{\beta}'_1, \boldsymbol{\beta}'_2, \ldots, \boldsymbol{\beta}'_d)'$ is a $dp \times 1$ vector of unknown regression coefficients.

The random-effects estimator for regression coefficients is

$$\widehat{\boldsymbol{\beta}}^* = \left(\sum_{j=1}^{K} \mathbf{X}'_j \mathbf{W}^*_j \mathbf{X}_j\right)^{-1} \sum_{j=1}^{K} \mathbf{X}'_j \mathbf{W}^*_j \widehat{\boldsymbol{\theta}}_j$$

where $\mathbf{W}^*_j = (\widehat{\boldsymbol{\Sigma}} + \boldsymbol{\Lambda}_j)^{-1}$. The corresponding covariance matrix is given by

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*) = \left(\sum_{j=1}^{K} \mathbf{X}'_j \mathbf{W}^*_j \mathbf{X}_j\right)^{-1} \tag{1}$$

Let $\mathbf{W}_j = \mathbf{\Lambda}_j^{-1}$. The fixed-effects estimator for regression coefficients is

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{j=1}^{K} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j \right)^{-1} \sum_{j=1}^{K} \mathbf{X}_j' \mathbf{W}_j \widehat{\boldsymbol{\theta}}_j$$

and the corresponding covariance matrix is

$$\text{Var}(\widehat{\boldsymbol{\beta}}) = \left(\sum_{j=1}^{K} \mathbf{X}_j' \mathbf{W}_j \mathbf{X}_j \right)^{-1} \tag{2}$$

Brief overview of heterogeneity statistics

estat heterogeneity supports the following heterogeneity statistics: Cochran I_Q^2 and H_Q^2, Jackson–White–Riley R_{JWR} and I_{JWR}^2, and White I_W^2. Some statistics can be computed separately for each outcome, some can be computed jointly for all outcomes, and some can be computed both jointly and separately. We summarize how each statistic can quantify heterogeneity in table 1, which is a modified version of Jackson, White, and Riley (2012, table V).

Table 1. Quantifying heterogeneity for various heterogeneity statistics

Statistic	Quantifies heterogeneity for		
	each outcome	all outcomes jointly	subsets of outcomes
I_Q^2, H_Q^2	No	Yes	No
I_{JWR}^2, R_{JWR}	Yes	Yes	Yes
I_W^2	Yes	No	No

Cochran heterogeneity statistics

The Cochran I_Q^2 and H_Q^2 are based on Q_{M}, a multivariate extension of the Cochran statistic Q_{res} defined in (3) in *Methods and formulas* of [META] **meta regress**. These statistics reduce to the I_{res}^2 and H_{res}^2 [see (4) and (5) in *Methods and formulas* of [META] **meta regress**] based on the DerSimonian–Laird random-effects method in the univariate case.

$$I_Q^2 = 100 \times \max\left(0, \frac{Q_{\text{M}} - N + dp}{Q_{\text{M}}}\right)$$

$$H_Q^2 = \max\left(1, \frac{Q_{\text{M}}}{N - dp}\right)$$

where $Q_{\text{M}} = \sum_{j=1}^{K} (\widehat{\boldsymbol{\theta}}_j - \mathbf{X}_j \widehat{\boldsymbol{\beta}})' \mathbf{W}_j (\widehat{\boldsymbol{\theta}}_j - \mathbf{X}_j \widehat{\boldsymbol{\beta}})$.

One drawback of the Cochran statistics is that they assess only the impact of heterogeneity jointly for all outcomes. It may be of interest to investigate separately the extent of each outcome's contribution to the total variability. This will be possible with all heterogeneity statistics discussed below.

Jackson–White–Riley heterogeneity statistics

Jackson, White, and Riley (2012) proposed to extend the heterogeneity statistic R of Higgins and Thompson (2002) (not to be confused with the R^2 statistic reported after [META] **meta regress**) to the multivariate setting. They then used the R statistic to construct an I^2 statistic that can quantify heterogeneity more conveniently. These I^2 and R statistics have an advantage over other versions of heterogeneity statistics because they may be computed for any subset of outcome variables.

For all outcomes jointly, the statistic R is defined as the ratio of the volumes of the confidence regions for the regression coefficients under the random-effects and fixed-effects models raised to the power $1/dp$,

$$R_{\mathrm{JWR}} = \left(\frac{\mathrm{Vol}_{\mathrm{RE}}}{\mathrm{Vol}_{\mathrm{FE}}}\right)^{1/dp}$$

where $\mathrm{Vol}_{\mathrm{RE}}$ and $\mathrm{Vol}_{\mathrm{FE}}$ are the volumes of the confidence regions under the random-effects and fixed-effects models, respectively. It can be shown (see Jackson, White, and Riley [2012, appendix]) that the volumes of these regions are proportional to the square root of the determinants of the covariance matrices defined in (1) and (2),

$$\mathrm{Vol}_{\mathrm{RE}} \propto \left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)\right|^{1/2} \quad \text{and} \quad \mathrm{Vol}_{\mathrm{FE}} \propto \left|\mathrm{Var}(\widehat{\boldsymbol{\beta}})\right|^{1/2}$$

In what follows, we will define the JWR R and I^2 statistics in terms of these determinants.

For all outcome jointly, the JWR R and I^2 statistics are given by

$$R_{\mathrm{JWR}} = \left\{\frac{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)\right|}{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}})\right|}\right\}^{1/2dp} = \left\{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)\right| \left|\mathrm{Var}(\widehat{\boldsymbol{\beta}})\right|^{-1}\right\}^{1/2dp}$$

$$I_{\mathrm{JWR}}^2 = 100 \times \max\left(0, \frac{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)\right|^{1/dp} - \left|\mathrm{Var}(\widehat{\boldsymbol{\beta}})\right|^{1/dp}}{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)\right|^{1/dp}}\right) = 100 \times \max\left(0, \frac{R_{\mathrm{JWR}}^2 - 1}{R_{\mathrm{JWR}}^2}\right)$$

where $\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)$ and $\mathrm{Var}(\widehat{\boldsymbol{\beta}})$ are the estimated covariance matrices of the regression coefficients under the random-effects and fixed-effects models, defined in (1) and (2), respectively.

For each outcome i, the expressions for the JWR statistics are

$$R_{\mathrm{JWR},\,i} = \left\{\left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i^*)\right| \left|\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i)\right|^{-1}\right\}^{1/2dp}$$

$$I_{\mathrm{JWR},\,i}^2 = 100 \times \max\left(0, \frac{R_{\mathrm{JWR},\,i}^2 - 1}{R_{\mathrm{JWR},\,i}^2}\right)$$

where $\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i^*)$ and $\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i)$ are the $p \times p$ submatrices of $\mathrm{Var}(\widehat{\boldsymbol{\beta}}^*)$ and $\mathrm{Var}(\widehat{\boldsymbol{\beta}})$ corresponding to the regression coefficient parameters of the ith outcome.

When the `jwriley(`*depnames*`)` option is specified, it is possible to compute the JWR statistics for any subset of outcomes defined by *depnames*. For a subset $\{s\}$ of outcome variables,

$$R_{\text{JWR},\,\{s\}} = \left\{ \left| \text{Var}(\widehat{\boldsymbol{\beta}}_{\{s\}}^{*}) \right| \, \left| \text{Var}(\widehat{\boldsymbol{\beta}}_{\{s\}}) \right|^{-1} \right\}^{1/2\nu}$$

$$I_{\text{JWR},\,\{s\}}^{2} = 100 \times \max\left(0, \, \frac{R_{\text{JWR},\,\{s\}}^{2} - 1}{R_{\text{JWR},\,\{s\}}^{2}} \right)$$

where $\widehat{\boldsymbol{\beta}}_{\{s\}}^{*}$ and $\widehat{\boldsymbol{\beta}}_{\{s\}}$ represent the set of regression coefficients corresponding to the subset $\{s\}$ and ν is the number of the corresponding estimated regression coefficients.

White heterogeneity statistics

White (2011) (option white) suggested to compute the I^2 statistic separately for each outcome but did not provide a method to quantify heterogeneity jointly based on all outcomes. For each outcome i, the White I^2 statistic is defined as

$$I_{\text{W},\,i}^{2} = \frac{\widehat{\boldsymbol{\Sigma}}_{ii}}{\widehat{\boldsymbol{\Sigma}}_{ii} + s_{\text{W},\,i}^{2}}$$

where $\widehat{\boldsymbol{\Sigma}}_{ii}$ is the ith diagonal element of the estimated between-study covariance matrix $\widehat{\boldsymbol{\Sigma}}$ described in *Iterative methods for computing* $\boldsymbol{\Sigma}$ and *Noniterative method for computing* $\boldsymbol{\Sigma}$ of [META] **meta mvregress**.

The typical within-study variance $s_{\text{W},\,i}^{2}$ is computed from a univariate meta-regression of the ith outcome on the moderators. Therefore, $s_{\text{W},\,i}^{2}$ is computed in the same way as s^2 in *Residual heterogeneity measures* in [META] **meta regress** and is given by

$$s_{\text{W},\,i}^{2} = \frac{n_i - p}{\text{tr}(\mathbf{P})}$$

where n_i is the number of observations on outcome i, which equals K if there are no missing values, and $\mathbf{P}$ is given by (1) in *Methods and formulas* of [META] **meta regress**, with $\mathbf{A} = \mathbf{W}$ being the $n_i \times n_i$ diagonal matrix containing inverse-variance weights for outcome i.

The $I_{\text{W},\,i}^{2}$ statistic reduces to I_{res}^{2} reported by (4) in *Methods and formulas* of [META] **meta regress** when $d = 1$.

References

Higgins, J. P. T., and S. G. Thompson. 2002. Quantifying heterogeneity in a meta-analysis. *Statistics in Medicine* 21: 1539–1558. https://doi.org/10.1002/sim.1186.

Jackson, D., I. R. White, and R. D. Riley. 2012. Quantifying the impact of between-study heterogeneity in multivariate meta-analyses. *Statistics in Medicine* 31: 3805–3820. https://doi.org/10.1002/sim.5453.

White, I. R. 2011. Multivariate random-effects meta-regression: Updates to mvmeta. *Stata Journal* 11: 240–254.

Also see

[META] **meta mvregress** — Multivariate meta-regression

[META] **meta mvregress postestimation** — Postestimation tools for meta mvregress

[U] **20 Estimation and postestimation commands**