Title

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hessenbergd( ) — Hessenberg decomposition

Description

`hessenbergd(`*A*`,` *H*`,` *Q*`)` calculates the Hessenberg decomposition of a square, numeric matrix, *A*, returning the upper Hessenberg form matrix in *H* and the orthogonal (unitary) matrix in *Q*. *Q* is orthogonal if *A* is real and unitary if *A* is complex.

`_hessenbergd(`*A*`,` *Q*`)` does the same as `hessenbergd()` except that it returns *H* in *A*.

`_hessenbergd_la()` is the interface to the LAPACK routines used to implement the above function; see [M-1] **LAPACK**. Its direct use is not recommended.

Syntax

void `hessenbergd(`*numeric matrix A*`,` *H*`,` *Q*`)`

void `_hessenbergd(`*numeric matrix A*`,` *Q*`)`

Remarks and examples

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The Hessenberg decomposition of a matrix, $\mathbf{A}$, can be written as

$$\mathbf{Q}' \times \mathbf{A} \times \mathbf{Q} = \mathbf{H}$$

where $\mathbf{H}$ is upper Hessenberg; $\mathbf{Q}$ is orthogonal if $\mathbf{A}$ is real or unitary if $\mathbf{A}$ is complex.

A matrix $\mathbf{H}$ is in upper Hessenberg form if all entries below its first subdiagonal are zero. For example, a 5×5 upper Hessenberg matrix looks like

```
        1   2   3   4   5
    +-------------------------+
  1 |  x   x   x   x   x      |
  2 |  x   x   x   x   x      |
  3 |  0   x   x   x   x      |
  4 |  0   0   x   x   x      |
  5 |  0   0   0   x   x      |
    +-------------------------+
```

For instance,

```
: A
        1    2    3    4    5
    +-------------------------+
  1 |  3    2    1   -2   -5  |
  2 |  4    2    1    0    3  |
  3 |  4    4    0    1   -1  |
  4 |  5    6    7   -2    4  |
  5 |  6    7    1    2   -1  |
    +-------------------------+

: hessenbergd(A, H=., Q=.)
```

```
: H
                 1              2              3              4              5
    +-------------------------------------------------------------------------+
  1 |             3    2.903464745   -.552977683   -4.78764119   -1.530555451 |
  2 | -9.643650761    7.806451613    2.878001755     5.1085876    5.580422694 |
  3 |             0   -3.454023879   -6.119229633   -.2347200215    1.467932097 |
  4 |             0              0    1.404136249   -1.715823624   -.9870601994 |
  5 |             0              0              0   -2.668128952    -.971398356 |
    +-------------------------------------------------------------------------+

: Q
                 1              2              3              4              5
    +-------------------------------------------------------------------------+
  1 |             1              0              0              0              0 |
  2 |             0   -.4147806779   -.0368006164   -.4047768558   -.8140997488 |
  3 |             0   -.4147806779   -.4871239484   -.5692309155    .5163752637 |
  4 |             0   -.5184758474    .8096135604   -.0748449196    .2647771074 |
  5 |             0   -.6221710168   -.3253949238    .7117092805   -.0221645995 |
    +-------------------------------------------------------------------------+
```

Many algorithms use a Hessenberg decomposition in the process of finding another decomposition with more structure.

Conformability

`hessenbergd(`*A*`,` *H*`,` *Q*`)`:

input:

A: $n \times n$

output:

H: $n \times n$

Q: $n \times n$

`_hessenbergd(`*A*`,` *Q*`)`:

input:

A: $n \times n$

output:

A: $n \times n$

Q: $n \times n$

Diagnostics

`_hessenbergd()` aborts with error if *A* is a view.

`hessenbergd()` and `_hessenbergd()` return missing results if *A* contains missing values.

Karl Adolf Hessenberg (1904–1959) was born in Frankfurt am Main, Germany. He was an electrical engineer and gained degrees from the Technische Hochschule Darmstadt. His doctoral dissertation, approved in 1942, was on computation of the eigenvalues and eigensolutions of linear systems of equations. In concurrent work, he introduced what are now called Hessenberg matrices. The mathematician Gerhard Hessenberg was a near relative.

Also see

[M-1] **LAPACK** — Linear algebra package (LAPACK) routines

[M-5] **schurd( )** — Schur decomposition

[M-4] **Matrix** — Matrix functions